Computation and Modeling Assignment 19

Anton Perez

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Problem 19-2

1. Part 1

   (a) Write the probability distribution $p_4(n)$ for getting n heads on 4 coin flips, where the coin is a fair coin (i.e. it lands on heads with probability 0.5)

   Solution:
   $$p_4(n) = \frac{4!}{2^4 n!(4-n)!} = \frac{3}{2n!(4-n)!}$$
   $p_4(0) = \frac{1}{16} = 0.0625$
   $p_4(1) = \frac{4}{16} = \frac{1}{4} = 0.25$
   $p_4(2) = \frac{6}{16} = \frac{3}{8} = 0.375$
   $p_4(3) = \frac{4}{16} = \frac{1}{4} = 0.25$
   $p_4(4) = \frac{1}{16} = 0.0625$
   Probability distribution: $[0.0625, 0.25, 0.375, 0.25, 0.0625]$

   (b) Let N be the number of heads in 4 coin flips. Then $N \sim p_4$. Intuitively, what is the expected value of N? Explain the reasoning behind your intuition.

   Solution: 2, because there's a 0.5 chance of getting heads, and therefore around half the coins flipped should end up heads.

   (c) Compute the expected value of N, using the definition
   $E[N] = \sum n \cdot p(n)$

   Solution:
   $$0(0.0625) + 1(0.25) + 2(0.375) + 3(0.25) + 4(0.0625) = 2$$

   (d) Compute the variance of N, using the definition
   $\text{Var}[N] = E[(N - E[N])^2]$

   Solution:
   $$(0-2)^2(0.0625) + (1-2)^2(0.25) + (2-2)^2(0.375) + (3-2)^2(0.25) + (4-2)^2(0.0625) = 1$$

2. Part 2

   (a) Write the probability distribution $p_{4,k}(n)$ for getting n heads on 4 coin flips, where the coin is a biased coin that lands on heads with probability k.

   Solution:
   $$p_{4,k}(n) = \frac{4!k^n(1-k)^{4-n}}{n!(4-n)!}$$
   Probability distribution: $[(1-k)^4, 4k(1-k)^3, 6k^2(1-k)^2, 4k^3(1-k), k^4]$

(b) Let N be the number of heads in 4 coin flips of a biased coin. Then $N \sim p_4, k$. Intuitively, what is the expected value of N? Your answer should be in terms of k. Explain the reasoning behind your intuition.

Solution: I'd expect it to be close to $4k$, because the probability of getting heads is k, and therefore around k of flips should be heads.

(c) Compute the expected value of N, using the definition $E[N] = \sum n \cdot p(n)$.

Solution:

$$0((1-k)^4) + 1(4k(1-k)^3) + 2(6k^2(1-k)^2) + 3(4k^3(1-k)) + 4(k^4)$$
$$= 4k(1-k)^3 + 12k^2(1-k)^2 + 12k^3(1-k) + 4k^4$$
$$= -4k^4 + 12k^3 - 12k^2 + 4k + 12k^4 - 24k^3 + 12k^2 + 12k^3 - 12k^4 + 4k^4$$
$$= 4k$$