Computation and Modeling Assignment 21

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Problem 21-1

A uniform distribution on the interval $[3, 7]$ is a probability distribution $p(x)$ that takes the following form for some constant k:

$$p(x) = \begin{cases} k & x \in [3, 7] \\ 0 & x \notin [3, 7] \end{cases}$$

It is also $\mathcal{U}[3, 7]$. So, to say that $X \sim \mathcal{U}[3, 7]$, is to say that $X \sim p$ for the function p shown above.

1. Find the value of k such that $p(x)$ is a valid probability distribution.

   Solution:

$$\begin{aligned} \int_{-\infty}^{\infty} p(x)\mathrm{d}x = \int_3^7 k\mathrm{d}x &= 1 \\ kx\Big|_3^7 &= 1 \\ 7k - 3k &= 1 \\ 4k &= 1 \\ k &= \frac{1}{4} \end{aligned}$$

2. Given that $X \sim \mathcal{U}[3, 7]$, compute $E[X]$.

   Solution:

$$\begin{aligned} \int_{-\infty}^{\infty} xp(x)\mathrm{d}x &= \int_3^7 kx\mathrm{d}x \\ &= \int_3^7 \frac{1}{4}x\mathrm{d}x \\ &= \frac{x^2}{8}\Big|_3^7 \\ &= \frac{49}{8} - \frac{9}{8} \\ &= 5 \end{aligned}$$

3. Given that $X \sim \mathcal{U}[3, 7]$, compute $\mathrm{Var}[X]$.

Solution:

$$\begin{aligned}\int_{-\infty}^{\infty} p(x)(x - E[X])^2 \mathrm{d}x &= \int_3^7 k(x - E[X])^2 \mathrm{d}x \\ &= \int_3^7 \frac{1}{4}(x-5)^2 \mathrm{d}x \\ &= \frac{(x-5)^3}{12}\Big|_3^7 \\ &= \frac{2}{3} - (-\frac{2}{3}) \\ &= \frac{4}{3}\end{aligned}$$