Computation and Modeling Assignment 26

Anton Perez

March 2, 2021

Problem 26-3

1. Part 1: Suppose that you take a bus to work every day. Bus A arrives at 8am but is x minutes late with $x \sim \mathcal{U}(0, 20)$. Bus B arrives at 8:10 but with $x \sim \mathcal{U}(0, 10)$. The bus ride is 20 minutes and you need to arrive at work by 8:30.

   (a) If you take bus A, what time do you expect to arrive at work? Justify your answer.

   Solution:

   $$\begin{aligned} E[X] &= \frac{a+b}{2} \\ &= \frac{0+20}{2} \\ &= 10 \text{ minutes} \end{aligned}$$

   I would expect to arrive at work at 8:30.

   (b) If you take bus B, what time do you expect to arrive at work? Justify your answer.

   Solution:

   $$\begin{aligned} E[X] &= \frac{a+b}{2} \\ &= \frac{0+10}{2} \\ &= 5 \text{ minutes} \end{aligned}$$

   I would expect to arrive at work at 8:35.

   (c) If you take bus A, what is the probability that you will arrive on time to work? Justify your answer.

   Solution: To still be on time, the bus can only be 10 minutes late.

   $$\begin{aligned} \int_0^{10} \frac{1}{b-a}\,\mathrm{d}x &= \int_0^{10} \frac{1}{20}\,\mathrm{d}x \\ &= \frac{x}{20}\Big|_0^{10} \\ &= \frac{1}{2} \end{aligned}$$

   The probability of being on time is $\frac{1}{2}$.

   (d) If you take bus B, what is the probability that you will arrive on time to work? Justify your answer.

Solution: To still be on time, the bus can't be late (or has to be 0 minutes late). The probability of arriving on time is 0.

2. Part 2: Continuing the scenario above, there is a third option that you can use to get to work: you can jump into a wormhole and (usually) come out almost instantly at the other side. The only issue is that time runs differently inside the wormhole, and while you're probably going to arrive at the other end very quickly, there's a small chance that you could get stuck in there for a really long time.

   The number of seconds it takes you to come out the other end of the wormhole follows an exponential distribution $\text{Exp}(\lambda = 4)$.

   (a) How long do you expect it to take you to come out of the wormhole? Justify your answer.

   Solution:

$$\begin{aligned} E[X] &= \frac{1}{\lambda} \\ &= \frac{1}{4} \text{ seconds} \end{aligned}$$

   (b) What's the probability of taking longer than a second to come out of the wormhole? Justify your answer.

   Solution:

$$\begin{aligned} \int_1^\infty \text{Exp}(\lambda = 4)\ \mathrm{d}x &= \int_1^\infty 4e^{-4x}\ \mathrm{d}x \\ &= -e^{-4x}\Big|_1^\infty \\ &= 0 - (-e^{-4}) \\ &= e^{-4} \\ &= 0.0183156 \end{aligned}$$

   (c) Fill in the blank: the probability of coming out of the wormhole within ___ seconds is 99.999%. Justify your answer.

   Solution:

$$\begin{aligned} \int_0^s 4e^{-4x}\ \mathrm{d}x &= 99.999\% \\ -e^{-4x}\Big|_0^s &= 0.99999 \\ -e^{-4s} + 1 &= 0.99999 \\ s &= 2.878231 \text{ seconds} \end{aligned}$$

   (d) Your friend says that you shouldn't use the wormhole because there's always a chance that you might get stuck in it for over a day, and if you use the wormhole often, then that'll probably happen sometime within your lifetime. Is this a reasonable fear? Why or why not? Justify your answer by computing the probability that you'll get stuck in the wormhole for over a day if you use the wormhole 10 times each day for 80 years.

   Solution:
   1 day = 86400 seconds

$$\begin{aligned} \int_{-\infty}^{86400} \text{Exp}(\lambda = 4)\ \mathrm{d}x &= \int_0^{86400} 4e^{-4x}\ \mathrm{d}x \\ &= -e^{-4x}\Big|_0^{86400} \\ &= 1 \end{aligned}$$

You will end up on the other side within the day with almost complete certainty. You will not get stuck in the wormhole over a day.