Computation and Modeling Assignment 30

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Problem 30-1

1. Let T be the time needed to complete a job at a certain factory. By using the historical data, we know that

$$P(T \leq t) = \begin{cases} \frac{1}{16}t^2 & \text{for } 0 \leq t \leq 4 \\ 1 & \text{for } t \geq 4 \end{cases}$$

   (a) Find the probability that the job is completed in less than one hour, i.e., find $P(T \leq 1)$

   Solution:

$$P(T \leq 1) = \frac{1}{16}$$

   (b) Find the probability that the job needs more than 2 hours.

   Solution:

$$\begin{aligned} P(T \geq 2) &= 1 - P(T \leq 2) \\ &= 1 - \frac{1}{4} \\ &= \frac{3}{4} \end{aligned}$$

   (c) Find the probability that $1 \leq T \leq 3$.

   Solution:

$$\begin{aligned} P(1 \leq T \leq 3) &= P(T \leq 3) - P(T \leq 1) \\ &= \frac{9}{16} - \frac{1}{16} \\ &= \frac{1}{2} \end{aligned}$$

2. You purchase a certain product. The manual states that the lifetime T of the product, defined as the amount of time (in years) the product works properly until it breaks down, satisfies

$$P(T \geq t) = e^{-\frac{t}{5}}, \text{ for all } t \geq 0$$

   What is the probability that it breaks down in the third year?

Solution:

$$\begin{aligned}
P(T \leq 3 | T \geq 2) &= \frac{P(2 \leq T \leq 3)}{P(T \geq 2)} \\
&= \frac{P(T \geq 2) - P(T \geq 3)}{P(T \geq 2)} \\
&= \frac{e^{-\frac{2}{5}} - e^{\frac{3}{5}}}{e^{-\frac{2}{5}}} \\
&= 0.1813
\end{aligned}$$

3. Consider a random experiment with a sample space,

$$S = \{1, 2, 3, ...\}$$

Suppose that we know

$$P(k) = P(\{k\}) = \frac{c}{3^k} \text{ for } k = 1, 2, ...$$

where c is a constant number.

(a) Find c.

Solution:

$$\begin{aligned}
\sum_{k=1}^{\infty} \frac{c}{3^k} &= 1 \\
c \sum_{k=1}^{\infty} \left(\frac{1}{3}\right)^k &= 1 \\
\frac{1}{2}c &= 1 \\
c &= 2
\end{aligned}$$

(b) Find $P(\{2, 4, 6\})$.

Solution:

$$\begin{aligned}
P(\{2, 4, 6\}) &= \frac{2}{3^2} + \frac{2}{3^4} + \frac{2}{3^6} \\
&= 0.249657
\end{aligned}$$

(c) Find $P(\{3, 4, 5, ...\})$

Solution:

$$\begin{aligned}
\sum_{k=3}^{\infty} \frac{2}{3^k} &= \sum_{k=1}^{\infty} \frac{2}{3^k} - \sum_{k=1}^{2} \frac{2}{3^k} \\
&= 1 - \left(\frac{2}{3} + \frac{2}{9}\right) \\
&= \frac{1}{9}
\end{aligned}$$