Computation and Modeling Assignment 31

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Problem 31-1

1. I roll a fair die twice and obtain two numbers: $X_1 =$ the result of the first roll, $X_2 =$ the result of the second roll.

   (a) Find the probability that $X_2 = 4$.

   Solution:

   $$P(X_2 = 4) = \frac{1}{6}$$

   (b) Find the probability that $X_1 + X_2 = 7$.

   Solution:

   $$P(X_1 + X_2 = 7) = \frac{6}{36} = \frac{1}{6}$$

   (c) Find the probability that $X_1 \neq 2$ and $X_2 \geq 4$.

   Solution:

   $$P(X_1 \neq 2) = 1 - P(X_1 = 2) = \frac{5}{6}$$

   $$P(X_1 \neq 2) = \frac{1}{2}$$

   $$\begin{aligned} P(X_1 \neq 2 \cap X_2 \geq 4) &= P(X_1 \neq 2)P(X_2 \geq 4) \\ &= \frac{5}{6} * \frac{1}{2} \\ &= \frac{5}{12} \end{aligned}$$

2. Let A and B be events such that

   $$P(A) = 0.4,\ P(B) = 0.7,\ P(A \cup B) = 0.9$$

   (a) $P(A \cap B)$

   Solution:

   $$\begin{aligned} P(A) + P(B) - P(A \cap B) &= P(A \cup B) \\ P(A \cap B) &= P(A) + P(B) - P(A \cup B) \\ P(A \cap B) &= 0.4 + 0.7 - 0.9 \\ P(A \cap B) &= 0.2 \end{aligned}$$

(b) $P(A^c \cap B)$

Solution:

$$P(A^c \cap B) = 0.5$$

(c) $P(A - B)$

Solution:

$$\begin{aligned} P(A - B) &= P(A) - P(A \cap B) \\ &= 0.4 - 0.2 \\ &= 0.2 \end{aligned}$$

(d) $P(A^c - B)$

Solution:

$$\begin{aligned} P(A^c - B) &= P(A^c) - P(A^c \cap B) \\ &= 0.6 - 0.5 \\ &= 0.1 \end{aligned}$$

(e) $P(A^c \cup B)$

Solution:

$$\begin{aligned} P(A^c \cup B) &= P(A^c) + P(B) - P(A^c \cap B) \\ &= 0.6 + 0.7 - 0.5 \\ &= 0.8 \end{aligned}$$

(f) $P(A \cap (B \cup A^c))$

Solution:

$$P(A \cap (B \cup A^c)) = 0.2$$

3. An urn contains 30 red balls and 70 green balls. What is the probability of getting exactly k red balls in a sample of size 20 if the sampling is done with replacement (repetition allowed)? Assume $0 \leq k \leq 20$.

Solution:

$$P(k) = {}_{20}C_k \left(\frac{3}{10}\right)^k \left(\frac{7}{10}\right)^{(20-k)}$$

4. An urn contains 30 red balls and 70 green balls. What is the probability of getting exactly k red balls in a sample of size 20 if the sampling is done without replacement (repetition not allowed)? Assume $0 \leq k \leq 20$.

Solution:

$$P(k) = {}_{30}P_k \; {}_{70}P_{20-k} \frac{80!}{100!} = \frac{30!70!80!}{100!(30-k)!(50-k)!}$$

5. Let X be a discrete random variable with the following PMF

$$P_X(x) = \begin{cases} 0.3 & \text{for } x = 3 \\ 0.2 & \text{for } x = 5 \\ 0.3 & \text{for } x = 8 \\ 0.2 & \text{for } x = 10 \\ 0 & \text{otherwise} \end{cases}$$

Find and plot the CDF of X.

Solution:

$$\text{CDF}(x) = \begin{cases} 0 & x < 3 \\ 0.3 & 3 \leq x < 5 \\ 0.5 & 5 \leq x < 8 \\ 0.8 & 8 \leq x < 10 \\ 1 & x \geq 10 \end{cases}$$