Computation and Modeling Assignment 42

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Problem 42-2

a) Let A, B and C be 3 events in the sample space S. Suppose we know

$$A \cup B \cup C = S$$
$$P(A) = \frac{1}{2}$$
$$P(B) = \frac{2}{3}$$
$$P(A \cup B) = \frac{5}{6}$$

Answer the following questions.

(a) Find $P(A \cap B)$.

Solution:

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) = \frac{1}{3}$$

(b) Do A, B and C form a partition of S?

Solution: No, because $P(A \cap B) \neq 0$.

(c) Find $P(C - (A \cup B))$.

Solution: Since $A \cup B \cup C = S$ and $P(S) = 1$

$$P(C - (A \cup B)) = 1 - P(A \cup B) = \frac{1}{6}$$

(d) If $P(C \cap (A \cup B)) = \frac{5}{12}$, find $P(C)$.

Solution:

$$P(C - (A \cup B)) = P(C) - P(C \cap (A \cup B))$$
$$P(C) = P(C - (A \cup B)) + P(C \cap (A \cup B)) = \frac{7}{12}$$

b) Let X and Y be two independent random variables. Suppose that we know $\text{Var}(2X - Y) = 6$ and $\text{Var}(X + 2Y) = 9$. Find $\text{Var}(X)$ and $\text{Var}(Y)$.

Solution: $\text{Cov}(X, Y) = 0$, since X and Y are independent variables.

$$\text{Var}(rX) = r^2\text{Var}(X)$$

$$\text{Var}(2X - Y) = 4\text{Var}(X) + \text{Var}(Y) = 6$$
$$\text{Var}(X + 2Y) = \text{Var}(X) + 4\text{Var}(Y) = 9$$

Combining both equations yields

$$-15\text{Var}(X) = -15$$
$$\text{Var}(X) = 1, \text{Var}(Y) = 2$$

c) Let X be a discrete random variable with the following PMF:

$$P_X(x) = \begin{cases} \frac{1}{2} & \text{for } x = 0 \\ \frac{1}{3} & \text{for } x = 1 \\ \frac{1}{6} & \text{for } x = 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find R_X, the range of the random variable X.

Solution:

$$R_X = \{0, 1, 2\}$$

(b) Find $P(X \geq 1.5)$.

Solution:

$$P(X \geq 1.5) = P_X(2) = \frac{1}{6}$$

(c) Find $P(0 < X < 2)$.

Solution:

$$P(0 < X < 2) = P_X(1) = \frac{1}{3}$$

(d) Find $P(x = 0|X < 2)$.

Solution:

$$P(X = 0|X < 2) = \frac{P(X = 0 \cap X < 2)}{P(X < 2)} = \frac{P_X(0)}{(P_X(0) + P_X(1))} = \frac{5}{12}$$

d) I roll two dice and observe two numbers X and Y. If $Z = X - Y$, find the range and PMF of Z.

Solution:

$$R_Z = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$$

$$P(z) = \begin{cases} \frac{1}{36} & \text{for } z = -5 \\ \frac{1}{18} & \text{for } z = -4 \\ \frac{1}{12} & \text{for } z = -3 \\ \frac{1}{9} & \text{for } z = -2 \\ \frac{5}{36} & \text{for } z = -1 \\ \frac{1}{6} & \text{for } z = 0 \\ \frac{5}{36} & \text{for } z = 1 \\ \frac{1}{9} & \text{for } z = 2 \\ \frac{1}{12} & \text{for } z = 3 \\ \frac{1}{18} & \text{for } z = 4 \\ \frac{1}{36} & \text{for } z = 5 \\ 0 & \text{otherwise} \end{cases}$$

e) Let A, B and C be 3 events with the probabilities given.

(a) Find $P(A|B)$.

Solution:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.35} = 0.5714$$

(b) Find $P(C|B)$.

Solution:

$$P(C|B) = \frac{P(C \cap B)}{P(B)} = \frac{0.15}{0.4} = 0.375$$

(c) Find $P(B|A \cup C)$.

Solution:

$$P(B|A \cup C) = \frac{P(B \cap (A \cup C))}{P(A \cup C)} = \frac{0.25}{0.7} = 0.3571$$

(d) Find $P(B|A, C) = P(B|A \cap C)$.

Solution:

$$P(B|A \cap C) = \frac{P(B \cap (A \cap C))}{P(A \cap C)} = \frac{0.1}{0.2} = 0.5$$

f) In a factory there are 100 units of a certain product, 5 of which are defective. We pick 3 units from the 100 units at random. What is the probability exactly one of them is defective?

Solution:

$$\frac{\binom{5}{1}\binom{95}{2}}{\binom{100}{3}} = 0.1381$$