Computation Modeling Assignment 20

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Problem 1

(a) *Using integration, show that this is a valid distribution, i.e. all the probability integrates to 1?*
(b) *Given that $X \sim p_2$, compute $P(0 < X \leq 1)$.*
You should get a result of $1 - e^{-2}$.
(c) *Given that $X \sim p_2$, compute $E[X]$.*
You should get a result of $\frac{1}{2}$.
(d) *Given that $X \sim p_2$, compute $Var[X]$.*
You should get a result of $\frac{1}{4}$.

Solution

(a) To show all the probabilities integrates to 1, when must show that $\int_{-\infty}^{\infty} p_2(x)\,\mathrm{d}x = 1$

$= \int_{-\infty}^{0} p_2(x)\,\mathrm{d}x + \int_{0}^{\infty} p_2(x)\,\mathrm{d}x$
$= \int_{-\infty}^{0} 0\,\mathrm{d}x + \int_{0}^{\infty} 2e^{-2x}\,\mathrm{d}x$
$= 0 - e^{-2\cdot\infty} + e^{-2\cdot 0}$
$= 1$

(b)

$= \int_{0}^{1} p_2(x)\,\mathrm{d}x$
$= \int_{0}^{1} 2e^{-2x}\,\mathrm{d}x$
$= -e^{-2\cdot 1} + e^{-2\cdot 0}$
$= 1 - e^{-2}$

(c)

$$\text{E[X]} = \int_{-\infty}^{\infty} x \cdot p_2(x)\,\mathrm{d}x$$
$$= \int_{-\infty}^{0} x \cdot 0\,\mathrm{d}x + \int_{0}^{\infty} 2xe^{-2x}\,\mathrm{d}x$$
$$= -\tfrac{1}{2} \cdot e^{-2\cdot\infty} \cdot (2 \cdot \infty + 1) + \tfrac{1}{2} \cdot e^{-2\cdot 0} \cdot (2 \cdot 0 + 1)$$
$$= \tfrac{1}{2}$$

(d)

$$\text{Var[X]} = \text{E[(X-E[X])}^2]$$
$$= \int_{-\infty}^{\infty} (x - \tfrac{1}{2})^2 \cdot p_2(x)\,\mathrm{d}x$$
$$= \int_{-\infty}^{0} (x - \tfrac{1}{2})^2 \cdot p_2(x)\,\mathrm{d}x + \int_{0}^{\infty} (x - \tfrac{1}{2})^2 \cdot p_2(x)\,\mathrm{d}x$$
$$= \int_{-\infty}^{0} (x - \tfrac{1}{2})^2 \cdot 0\,\mathrm{d}x + \int_{0}^{\infty} (x - \tfrac{1}{2})^2 \cdot 2e^{-2x}\,\mathrm{d}x$$
$$= \int_{0}^{\infty} 2e^{-2x} \cdot (x - \tfrac{1}{2})^2\,\mathrm{d}x$$
$$= -e^{-2\cdot\infty} \cdot \infty^2 - \tfrac{1}{4}e^{-2\cdot\infty} - (-e^{-2\cdot 0} \cdot 0^2 - \tfrac{1}{4}e^{-2\cdot 0})$$
$$= \tfrac{1}{4}$$