Computation Modeling Assignment 24

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Problem 24-2

(a) *Compute the likelihood of the observed outcome if the coin were fair (i.e.* $k = 0.5$ *). SHOW YOUR WORK and round your answer to 5 decimal places.*

$$P(\text{HHTTH} \mid k = 0.5) = P(\text{H} \mid k = 0.5) \cdot P(\text{H} \mid k = 0.5) \cdot P(\text{T} \mid k = 0.5) \cdot P(\text{T} \mid k = 0.5) \cdot P(\text{H} \mid k = 0.5)$$
$$= ?$$

(b) *Compute the likelihood of the observed outcome if the coin were slightly biased towards heads, say* $p = 0.55$*. SHOW YOUR WORK and round your answer to 5 decimal places.*

$$P(\text{HHTTH} \mid k = 0.55) = P(\text{H} \mid k = 0.55) \cdot P(\text{H} \mid k = 0.55) \cdot P(\text{T} \mid k = 0.55) \cdot P(\text{T} \mid k = 0.55) \cdot P(\text{H} \mid k = 0.55)$$
$$= ?$$

(c) *Compute the likelihood of the observed outcome for a general value of* p*. Your answer should be a function of* k*.*

$$P(\text{HHTTH} \mid k) = P(\text{H} \mid k) \cdot P(\text{H} \mid k) \cdot P(\text{T} \mid k) \cdot P(\text{T} \mid k) \cdot P(\text{H} \mid k)$$
$$= ?$$

Check: When you plug in $k = 0.5$, you should get the answer from part (a), and when you plug in $k = 0.55$, you should get the answer from part (b).

(d) *Plot a graph of* $P(HHTTH|k)$ *for* $0 \leq k \leq 1$*, and include the graph in your writeup.*
Note: You can use the same plotting code as usual. You'll just need to come up with a list of many data points on the function $y = P(HHTTH|k)$

Solution

(a)

$$\begin{aligned} P(\mathrm{HHTTH} \mid k = 0.5) &= P(\mathrm{H} \mid k = 0.5) \cdot P(\mathrm{H} \mid k = 0.5) \cdot P(\mathrm{T} \mid k = 0.5) \cdot P(\mathrm{T} \mid k = 0.5) \cdot P(\mathrm{H} \mid k = 0.5) \\ &= 0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 \\ &= 0.3125 \end{aligned}$$

(b)

$$\begin{aligned} P(\mathrm{HHTTH} \mid k = 0.55) &= P(\mathrm{H} \mid k = 0.55) \cdot P(\mathrm{H} \mid k = 0.55) \cdot P(\mathrm{T} \mid k = 0.55) \cdot P(\mathrm{T} \mid k = 0.55) \cdot P(\mathrm{H} \mid k = 0.55) \\ &= 0.55 \cdot 0.55 \cdot 0.45 \cdot 0.45 \cdot 0.55 \\ &= 0.03369 \end{aligned}$$

(c)

$$\begin{aligned} P(\mathrm{HHTTH} \mid k) &= P(\mathrm{H} \mid k) \cdot P(\mathrm{H} \mid k) \cdot P(\mathrm{T} \mid k) \cdot P(\mathrm{T} \mid k) \cdot P(\mathrm{H} \mid k) \\ &= k \cdot k \cdot (1-k) \cdot (1-k) \cdot k \\ &= k^3 \cdot (1-k)^2 \\ &= k^5 \cdot (k^2 - 2k + 1) \\ &= k^5 - 2k^4 + k^3 \end{aligned}$$

(d)