Computation Modeling Assignment 26

Your Name

March 2, 2021

Problem 26-3 Part 1

(a) *If you take bus A, what time do you expect to arrive at work? Justify your answer.*
(b) *If you take bus B, what time do you expect to arrive at work? Justify your answer.*
(c) *If you take bus A, what is the probability that you will arrive on time to work? Justify your answer.*
(d) *If you take bus B, what is the probability that you will arrive on time to work? Justify your answer.*

Solutions to Part 1

(a) We get to work at 8:00 + E[X] + 20 minutes

$$\begin{aligned} \mathrm{E}[X] &= \frac{a+b}{2} \\ &= \frac{0+20}{2} \\ &= 10 \end{aligned}$$

Taking bus A, we expect to arrive at 8:30

(b) We get to work at 8:10 + E[X] + 20 minutes

$$\begin{aligned} \mathrm{E}[X] &= \frac{a+b}{2} \\ &= \frac{0+10}{2} \\ &= 5 \end{aligned}$$

Taking bus B, we expect to arrive at 8:35

(c) For us to arrive at work on time, the bus can be 10 minutes late max. To find the probability we get to work on time, we must calculate the probability the bus will be at most 10 minutes late.

$$\begin{aligned} P(x \leq 10) &= \int_0^{10} p(x) \ \mathrm{d}x \\ &= \int_0^{10} \frac{1}{20} \ \mathrm{d}x \\ &= \frac{x}{20}\Big|_0^{10} \\ &= \frac{1}{2} \end{aligned}$$

(d) For us to arrive at work on time, the bus can be 0 minutes late max. To find the probability we get to work on time, we must calculate the probability the bus will be at most 0 minutes late.

$$\begin{aligned} P(x \leq 0) &= \int_0^{0} p(x) \ \mathrm{d}x \\ &= 0 \end{aligned}$$

Problem 26-3 Part 2

(a) *How long do you expect it to take you to come out of the wormhole? Justify your answer.*
(b) *What's the probability of taking longer than a second to come out of the wormhole? Justify your answer.*
(c) *Fill in the blank: the probability of coming out of the wormhole within _ seconds is* 99.999%. *Justify your answer.*
(d) *Your friend says that you shouldn't use the wormhole because there's always a chance that you might get stuck in it for over a day, and if you use the wormhole often, then that'll probably happen sometime within your lifetime. Is this a reasonable fear? Why or why not? Justify your answer by computing the probability that you'll get stuck in the wormhole for over a day if you use the wormhole 10 times each day for 80 years.*

Hint: It's easier to start by computing the probability that you won't get stuck in the wormhole for over a day on any given trip through the wormhole, and then use that to compute the probability that you won't get stuck in the wormhole for over a day if you use the wormhole 10 times each day for 80 years.

Solutions to Part 2

(a)

$$\begin{aligned} \mathrm{E}[X] &= \frac{1}{\lambda} \\ &= \frac{1}{4} \\ &= 0.25 \end{aligned}$$

We are expected to come out of the wormhole in about 0.25 seconds

(b)

$$\begin{aligned} P(x > 1) &= \int_1^\infty p_4(x) \ \mathrm{d}x \\ &= \int_1^\infty 4e^{-4x} \ \mathrm{d}x \\ &= -e^{-4x}\big|_1^\infty \\ &= -e^{-4\cdot\infty} + e^{-4} \\ &= 0.0183 \end{aligned}$$

(c)

$$\begin{aligned} 99.999\% &= \int_0^n 4e^{-4x} \ \mathrm{d}x \\ 0.99999 &= -e^{-4x}\big|_0^n \\ 0.99999 &= -e^{-4n} + 1 \\ e^{-4n} &= 0.00001 \\ \ln(e^{-4n}) &= \ln(0.00001) \\ -4n &= -11.51292546 \\ n &= 2.878231365 \\ &\approx 2.88 \end{aligned}$$

(d) 1 day = 86400 seconds. In 80 years, we use the wormhole 292000 times.

$$\begin{aligned} \text{chances of not getting stuck for one use} &= \int_0^{86400} 4e^{-4x} \ \mathrm{d}x \\ &= 1 - e^{-345600} \\ \text{chances when using 10 times a day for 80 years} &= \left(1 - e^{-345600}\right)^{10\cdot 80\cdot 365\cdot 86400} \\ &\approx 1 \end{aligned}$$