Computation Modeling Assignment 42

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Problem 42-1

a[a]

$$\begin{aligned} P(A \cap B) &= P(A) + P(B) - P(A \cup B) \\ &= \frac{1}{2} + \frac{2}{3} - \frac{5}{6} \\ &= \frac{1}{3} \end{aligned}$$

a[b] Because the intersection between A and B isn't 0, A, B, C do not form a partition of S.

a[c]

$$\begin{aligned} P(C - (A \cup B)) &= \text{Things only in } C \\ &= 1 - \left(\frac{1}{2} - \frac{1}{3} - (A \cap C) + (A \cap C) + \frac{1}{3} + \frac{2}{3} - \frac{1}{3} - (C \cap B) + (C \cap B)\right) \\ &= 1 - \left(\frac{1}{2} + \frac{1}{3}\right) \\ &= \frac{1}{6} \end{aligned}$$

a[d]

$$\begin{aligned} P(C) &= \frac{1}{6} + \frac{5}{12} \\ &= \frac{7}{12} \end{aligned}$$

b

$$\begin{aligned}\mathrm{Var}[2X-Y] &= 6\\ \mathrm{Var}[X+2Y] &= 9\end{aligned}$$

$$\begin{aligned}\mathrm{Var}[2X]+\mathrm{Var}[-Y]+2\mathrm{Cov}[2X,-Y] &= 6\\ \mathrm{Var}[X]+\mathrm{Var}[2Y]+2\mathrm{Cov}[X,2Y] &= 9\end{aligned}$$

$$\begin{aligned}\mathrm{Var}[2X]+\mathrm{Var}[-Y] &= 6\\ \mathrm{Var}[X]+\mathrm{Var}[2Y] &= 9\end{aligned}$$

$$\begin{aligned}4\mathrm{Var}[X]+1\mathrm{Var}[Y] &= 6\\ \mathrm{Var}[X]+4\mathrm{Var}[Y] &= 9\end{aligned}$$

$$\begin{aligned}\mathrm{Var}[Y]-16\mathrm{Var}[Y] &= -30\\ -15\mathrm{Var}[Y] &= -30\\ \mathrm{Var}[Y] &= 2\end{aligned}$$

$$\begin{aligned}4\mathrm{Var}[X]+2 &= 6\\ \mathrm{Var}[X] &= 1\end{aligned}$$

c[a]

$$R_X \in \{0,1,2\}$$

c[b]

$$\begin{aligned}P(X \geq 1.5) &= P(2)\\ &= \frac{1}{6}\end{aligned}$$

c[c]

$$\begin{aligned}P(0 < X < 2) &= P(1)\\ &= \frac{1}{3}\end{aligned}$$

c[d]

$$
\begin{aligned}
P(X=0 \mid X<2) &= \frac{P(X=0 \text{ and } X<2)}{P(X<2)} \\
&= \frac{\frac{1}{2}}{\frac{1}{2}+\frac{1}{3}} \\
&= \frac{3}{3+2} \\
&= \frac{3}{5}
\end{aligned}
$$

d

$$
P(z) = \begin{cases}
\frac{1}{36} & , z=-5 \\
\frac{1}{18} & , z=-4 \\
\frac{1}{12} & , z=-3 \\
\frac{1}{9} & , z=-2 \\
\frac{5}{36} & , z=-1 \\
\frac{1}{6} & , z=0 \\
\frac{5}{36} & , z=1 \\
\frac{1}{9} & , z=2 \\
\frac{1}{12} & , z=3 \\
\frac{1}{18} & , z=4 \\
\frac{1}{36} & , z=5
\end{cases}
$$

e[a]

$$
\begin{aligned}
P(A \mid B) &= \frac{P(A \text{ and } B)}{P(B)} \\
&= \frac{0.2}{0.35} \\
&= 0.5714
\end{aligned}
$$

e[b]

$$
\begin{aligned}
P(C \mid B) &= \frac{P(C \text{ and } B)}{P(B)} \\
&= \frac{0.15}{0.35} \\
&= 0.4286
\end{aligned}
$$

e[c]

$$\begin{aligned} P(B \mid A \cup C) &= \frac{P(B \text{ and } A \cup C)}{P(A \cup C)} \\ &= \frac{0.25}{0.7} \\ &= 0.3571 \end{aligned}$$

e[d]

$$\begin{aligned} P(B \mid A \cap C) &= \frac{P(B \text{ and } A \cap C)}{P(A \cap C)} \\ &= \frac{0.1}{0.2} \\ &= 0.5 \end{aligned}$$

f

$$\begin{aligned} \text{Probability exactly 1 is defective} &= \frac{5}{100} \cdot \frac{95}{99} \cdot \frac{94}{98} \cdot \binom{3}{1} \\ &= \frac{44650}{970200} \cdot 3 \\ &= 0.1381 \end{aligned}$$