Eurisko Assignment 24-2

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Suppose we have a coin that lands on heads with probability k and tails with probability $1 - k$. We flip the coin 5 times and get $HHTTH$.

(a) Compute the likelihood of the observed outcome if the coin were fair (i.e. $k = 0.5$)

$$\begin{aligned} P(\mathrm{HHTTH} \mid k = 0.5) &= P(\mathrm{H} \mid k = 0.5) \cdot P(\mathrm{H} \mid k = 0.5) \cdot P(\mathrm{T} \mid k = 0.5) \cdot P(\mathrm{T} \mid k = 0.5) \cdot P(\mathrm{H} \mid k = 0.5) \\ &= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \\ &= \frac{1}{2^5} \\ &= \frac{1}{32} \\ &= 0.03125 \end{aligned}$$

(b) Compute the likelihood of the observed outcome if the coin were slightly biased towards heads, say $k = 0.55$.

We know that the probability of getting n in f coin flips with a biased coin of probability k is

$$\frac{f!}{n! \cdot (f - n)!} \cdot k^n \cdot (1 - k)^{f - n}$$

Since n and p are both 1, we find the probability of getting 1 head in 1 coin flip with biased probability k is now

$$P(\mathrm{H} \mid k) = \frac{1!}{1! \cdot (1 - 1)!} \cdot k^1 \cdot (1 - k)^{1-1} = \frac{1}{1} \cdot k^1 \cdot (1 - k)^{1-1} = k$$

Also, the probability of getting 1 tail in 1 coin flip with biased probability k is now

$$P(\mathrm{T} \,|\, k) = \frac{1!}{1! \cdot (1-1)!} \cdot (1-k)^1 \cdot (1-(1-k))^{1-1} = \frac{1}{1} \cdot (1-k) \cdot 1 = 1-k$$

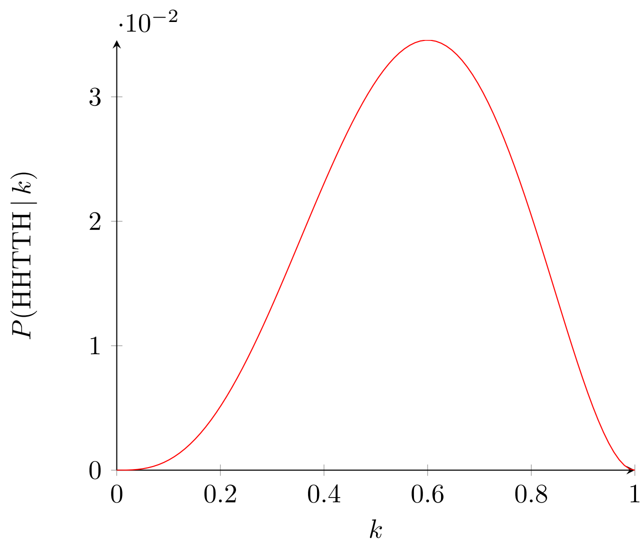
We can use these to solve for $P(\mathrm{HHTTH} \,|\, k = 0.55)$:

$$\begin{aligned} P(\mathrm{HHTTH} \,|\, k = 0.55) &= P(\mathrm{H} \,|\, k = 0.55) \cdot P(\mathrm{H} \,|\, k = 0.55) \cdot P(\mathrm{T} \,|\, k = 0.55) \cdot P(\mathrm{T} \,|\, k = 0.55) \cdot P(\mathrm{H} \,|\, k = 0.55) \\ &= \Big(P(\mathrm{H} \,|\, k = 0.55)\Big)^3 \cdot \Big(P(\mathrm{T} \,|\, k = 0.55)\Big)^2 \\ &= (k)^3 \cdot (1-k)^2 \\ &= (0.55)^3 \cdot (0.45)^2 \\ &= 0.166375 + 0.2025 \\ &= 0.368875 \end{aligned}$$

(c) Compute the likelihood of the observed outcome for a general value of p. Your answer should be a function of k.

$$\begin{aligned} P(\mathrm{HHTTH} \,|\, k) &= P(\mathrm{H} \,|\, k) \cdot P(\mathrm{H} \,|\, k) \cdot P(\mathrm{T} \,|\, k) \cdot P(\mathrm{T} \,|\, k) \cdot P(\mathrm{H} \,|\, k) \\ &= \Big(P(\mathrm{H} \,|\, k)\Big)^3 \cdot \Big(P(\mathrm{T} \,|\, k)\Big)^2 \\ &= k^3 \cdot (1-k)^2 \end{aligned}$$

(d) Plot a graph of $P(\mathrm{HHTTH} \,|\, k)$ for $0 \leq k \leq 1$, and include the graph in your writeup.



$\cdot 10^{-2}$
$P(\text{HHTTH} \mid k)$
3
2
1
0
0
0.2
0.4
0.6
0.8
1
k