Eurisko Assignment 30-1

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(a) Let T be the time needed to complete a job at a certain factory. By using historical data, we know that

$$P(T \leq t) = \begin{cases} \frac{t^2}{16} & 0 \leq t \leq 4 \\ 1 & t \geq 4 \end{cases}$$

(I) Find the probability that the job in completed in less than an hour, i.e. find $P(T \leq 1)$.

$$P(T \leq 1) = \frac{1^2}{16} = \frac{1}{16}$$

(II) Find the probability that the job needs more than 2 hours.

$$P(2 \leq T) = 1 - P(T \leq 2) = 1 - \frac{2^2}{16} = 1 - \frac{1}{4} = \frac{3}{4}$$

(III) Find the probability $1 \leq T \leq 3$.

$$P(1 \leq T \leq 3) = P(T \leq 3) - P(T \leq 1) = \frac{3^2}{16} - \frac{1^2}{16} = \frac{1}{2}$$

(b) You purchase a certain product. The manual states that the lifetime T of the product, defined as the amount of time (in years) the product works properly until it breaks down, satisfies

$$P(T \geq t) = e^{-\frac{t}{5}}, t \geq 0.$$

For example, the probability that the product lasts more than (or equal to) 2 years is $P(T \geq 2) = e^{-\frac{2}{5}} = 0.6703$. I purchase the product and use it for two years without any problems. What is the probability that it breaks down in the third year?

$$
\begin{aligned}
P(T \leq 3 \mid T \geq 2) &= \frac{P(2 \geq T \geq 3)}{P(T \leq 2)} \\
&= \frac{P(2 \leq T) - P(3 \geq T)}{P(T \leq 2)} \\
&= \frac{e^{-\frac{2}{5}} - e^{-\frac{3}{5}}}{e^{-\frac{2}{5}}} \\
&= 0.181269247
\end{aligned}
$$

(c) Consider the random experiment with a sample sequence $S = \{1, 2, 3, \dots\}$. Suppose we know

$$
P(k) = P(\{k\}) = \frac{c}{3^k}, k = 1, 2, \dots
$$

where c is a constant number.

(l) Find c.

$$
\begin{aligned}
\sum_{k=1}^{\infty} \frac{c}{3^k} &= 1 \\
c \cdot \sum_{k=1}^{\infty} \frac{1}{3^k} &= 1 \\
c \cdot \sum_{k=1}^{\infty} \left(\frac{1}{3}\right)^k &= 1 \\
\frac{c}{3(1 - \frac{1}{3})} &= 1 \\
c &= 2
\end{aligned}
$$

(ll) Find $P(\{2, 4, 6\})$.

$$
P(\{2, 4, 6\}) = \frac{2}{3^2} + \frac{2}{3^4} + \frac{2}{3^6} = 0.249657064
$$

(lll) Find $P(\{3, 4, 5, \dots\})$

$$\begin{aligned}
\sum_{k=3}^{\infty} \frac{2}{3^k} &= \sum_{k=1}^{\infty} \frac{2}{3^k} - \sum_{k=1}^{2} \frac{2}{3^k} \\
&= 1 - \left(\frac{2}{3} + \frac{2}{9}\right) \\
&= \frac{1}{9}
\end{aligned}$$