Eurisko Assignment 31-1

Charlie Weinberger

April 7, 2021

(a) I roll a fair die twice and obtain two numbers: X_1 as a result of the first roll, and X_2 as a result of the second roll.

(I) Find the probability that $X_2 = 4$.

$$P(x_2 = 4) = \frac{1}{6}$$

(II) Find the probability that $X_1 + X_2 = 7$.

$$\begin{aligned} P(x_1 + X_2 = 7) &= P(x_1 = 1 \text{ and } X_2 = 6) + P(x_1 = 2 \text{ and } X_2 = 5) + P(x_1 = 3 \text{ and } X_2 = 4) \\ &+ P(x_1 = 4 \text{ and } X_2 = 3) + P(x_1 = 5 \text{ and } X_2 = 2) + P(x_1 = 6 \text{ and } X_2 = 1) \\ &= \left(\frac{1}{6}\right)^2 + \left(\frac{1}{6}\right)^2 + \left(\frac{1}{6}\right)^2 + \left(\frac{1}{6}\right)^2 + \left(\frac{1}{6}\right)^2 + \left(\frac{1}{6}\right)^2 \\ &= 6 \cdot \left(\frac{1}{6}\right)^2 \\ &= \frac{1}{6} \end{aligned}$$

x

(III) Find the probability that $X_1 \neq 2$ and $X_2 \geq 4$.

$$
\begin{aligned}
P(X_1 \neq 2 \text{ and } X_2 \geq 4) &= (P(x_1 = 1 \text{ or } x_1 = 3 \text{ or } x_1 = 4 \text{ or } x_1 = 5 \text{ or } x_1 = 6)) \cdot \\
&\quad (P(x_2 = 4 \text{ or } x_2 = 5 \text{ or } x_2 = 6)) \\
&= \left(\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}\right) \cdot \left(\frac{1}{6} + \frac{1}{6} + \frac{1}{6}\right) \\
&= \left(\frac{5}{6}\right) \cdot \left(\frac{3}{6}\right) \\
&= \frac{15}{6} = 2.5
\end{aligned}
$$

(b) Let A and B be two events such that

$$P(A) = 0.4, P(B) = 0.7, P(A \cup B) = 0.9$$

(I) Find $P(A \cap B)$.

$$
\begin{aligned}
P(A \cap B) &= P(A) + P(B) - P(A \cup B) \\
&= 0.4 + 0.7 - 0.9 \\
&= 0.2
\end{aligned}
$$

(II) Find $P(A^c \cap B)$

$$
\begin{aligned}
P(A^c \cap B) &= P(B - A) \\
&= P(B) - P(B \cap A) \\
&= 0.7 - 0.2 \\
&= 0.5
\end{aligned}
$$

(III) Find $P(A - B)$

$$
\begin{aligned}
P(A - B) &= P(A) - P(A \cap B) \\
&= 0.4 - 0.2 \\
&= 0.2
\end{aligned}
$$

(IV) Find $P(A^c - B)$

$$
\begin{aligned}
P(A^c - B) &= P(A^c) - P(A^c \cap B) \\
&= (1 - P(A)) - 0.5 \\
&= 1 - 0.4 - 0.5 \\
&= 0.1
\end{aligned}
$$

(V) Find $P(A^c \cup B)$

$$
\begin{aligned}
P(A^c \cup B) &= P(A^c) + P(B) - P(A^c \cap B) \\
&= (1 - P(A)) + 0.7 - 0.5 \\
&= 1 - 0.4 + 0.7 - 0.5 \\
&= 0.8
\end{aligned}
$$

(VI) Find $P(A \cap (B \cup A^c))$

$$
\begin{aligned}
P(A \cap (B \cup A^c)) &= P((A \cap B) \cup (A \cap A^c)) \\
&= P(A \cap B) \\
&= 0.2
\end{aligned}
$$

(c) An urn contains 30 red balls and 70 green balls. What is the probability of getting exactly k red balls in a sample of size 20 if the sampling is done **with** replacement (repetition allowed)? Assume $0 \leq k \leq 20$.

$$
\frac{20!}{k! \cdot (20-k)!} \cdot (0.3)^k \cdot (0.7)^{20-k}
$$

(d) An urn contains 30 red balls and 70 green balls. What is the probability of getting exactly k red balls in a sample of size 20 if the sampling is done **without** replacement (repetition not allowed)?

$$
\frac{\dfrac{20!}{k! \cdot (20-k)!} + \dfrac{70!}{(20-k)! \cdot (50+k)!}}{\dfrac{100!}{20! \cdot (100-20)!}}
$$

(e) Let x be a discrete random variable with the following probability mass function (PMF)

$$P_X(x) = \begin{cases} 0.3 & \text{for } x = 3 \\ 0.2 & \text{for } x = 5 \\ 0.3 & \text{for } x = 8 \\ 0.2 & \text{for } x = 10 \\ 0 & \text{otherwise} \end{cases}$$

Find and plot the cumulative distribution function (CDF).