Eurisko Assignment 36-1

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Suppose you are a mission control analyst who is looking down at an enemy headquarters through a satellite view, and you want to get an estimate of how many tanks they have. Most of the headquarters is hidden, but you notice that near the entrance, there are four tanks visible, and these tanks are labeled with the numbers $52, 30, 68, 7$. So, you assume that they have N tanks that they have labeled with numbers from 1 to N.

Your commander asks you for an estimate: **with 95% certainty, what's the max number of tanks they have?**

In this problem, you'll answer that question using the same process that you used in 35-1 (a,b,f). In your answer, show your work and justify every step of the way.

Solution:

Because we are using the same process as 35-1 to answer to question, we know that our probability function must be

$$p_k(x) = \begin{cases} \frac{1}{k} & x \in \{1, 2, \ldots, k\} \\ 0 & x \notin \{1, 2, \ldots, k\} \end{cases}$$

Multiplying $\frac{1}{k}$ by itself four times (once for each number), we see that the prior distribution is

$$P(\{52, 30, 68, 7\} \mid k) = \begin{cases} \dfrac{1}{k^4} & k \geq 68 \\ 0 & \text{otherwise} \end{cases}$$

To find the posterior distribution, we must first calculate the value of c such that

$$\sum_{k=1}^{\infty} c \cdot P(\{1, 17, 8, 25, 3\} \mid k) = 1$$

We take that equation and solve for c:

$$\sum_{k=1}^{\infty} c \cdot P(\{52, 30, 68, 7\} \mid k) = 1$$
$$c \cdot \sum_{k=1}^{\infty} P(\{52, 30, 68, 7\} \mid k) = 1$$
$$c = \frac{1}{\sum_{k=1}^{\infty} P(\{52, 30, 68, 7\} \mid k)}$$
$$c = 922742.1505$$

Now, we can plug c in and find the posterior distribution:

$$P(k \mid \{52, 30, 68, 7\}) = c \cdot P(\{52, 30, 68, 7\} \mid k)$$
$$P(k \mid \{52, 30, 68, 7\}) = \begin{cases} \frac{922742.1505}{k^4} & k \geq 68 \\ 0 & \text{otherwise} \end{cases}$$

Using the code that I wrote for 35-1, I can be 95% sure that the upper bound is less than 183.