Eurisko Assignment 42-2

Charlie Weinberger

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(a) Let A, B, and C be three events in the simple space S. Suppose we know that:

$$- A \cup B \cup C = S,$$
$$- P(A) = \frac{1}{2},$$
$$- P(B) = \frac{2}{3},$$
$$- P(A \cup B) = \frac{5}{6}$$

(I) $P(A \cap B) = P(A) + P(B) - P(A \cup B) = \frac{1}{2} + \frac{2}{3} - \frac{5}{6} = \frac{1}{3}$

(II) No, because A and B overlap. We know this because the intersection of A and B is not 0.

(III) $P(C - (A \cup B)) = P((A \cup B)^c) = 1 - P(A \cup B) = 1 - \frac{5}{6} = \frac{1}{6}$

(IV) $P(C) = P(C \cap (A \cup B)) + P((A \cup B)^c) = \frac{5}{12} + \frac{1}{6} = \frac{7}{12}$

(b)

$$\begin{aligned} Var[2X - Y] &= Var[2X] + Var[-Y] + 2Cov[2X, -Y] \\ &= 4Var[X] + Var[Y] - 4Cov[X, Y] \\ &= 4Var[X] + Var[Y] = 6 \end{aligned}$$

$$\begin{aligned} Var[X + 2Y] &= Var[X] + Var[2Y] + 2Cov[X, 2Y] \\ &= Var[X] + 4Var[Y] + 4Cov[X, Y] \\ &= Var[X] + 4Var[Y] = 9 \end{aligned}$$

$$\left[\begin{array}{cc|c} 4 & 1 & 6 \\ 1 & 4 & 9 \end{array}\right] = \left[\begin{array}{cc|c} 1 & 4 & 9 \\ 4 & 1 & 6 \end{array}\right] = \left[\begin{array}{cc|c} 1 & 4 & 9 \\ 0 & 15 & 30 \end{array}\right] = \left[\begin{array}{cc|c} 1 & 4 & 9 \\ 0 & 1 & 2 \end{array}\right] = \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 2 \end{array}\right]$$

$$\boxed{Var[X] = 1, Var[Y] = 2}$$

(c) Let X be a discrete random variable with the following PMF:

$$P_X(x) = \begin{cases} \frac{1}{2} & \text{for } x = 0 \\ \frac{1}{3} & \text{for } x = 1 \\ \frac{1}{6} & \text{for } x = 2 \\ 0 & \text{otherwise} \end{cases}$$

(I) $R_X = 2 - 0 = 2$

(II) $P(X \geq 1.5) = P(X = 2) = \dfrac{1}{6}$

(III) $P(0 < X < 2) = P(X = 1) = \dfrac{1}{3}$

(IV) $P(X = 0 \mid X < 2) = \dfrac{P(X = 0)}{P(X = 0) + P(X = 1)} = \dfrac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{3}} = \dfrac{3}{5}$

(d) I roll two dice and observe two numbers X and Y. If $Z = X - Y$, find the range and PMF of Z.

(I) $R_X = 5 - -5 = 10$

(II)

$$P_Z(z) = \begin{cases} \frac{1}{36} & \text{for } z = -5 \\ \frac{2}{36} & \text{for } z = -4 \\ \frac{3}{36} & \text{for } z = -3 \\ \frac{4}{36} & \text{for } z = -2 \\ \frac{5}{36} & \text{for } z = -1 \\ \frac{6}{36} & \text{for } z = 0 \\ \frac{5}{36} & \text{for } z = 1 \\ \frac{4}{36} & \text{for } z = 2 \\ \frac{3}{36} & \text{for } z = 3 \\ \frac{2}{36} & \text{for } z = 4 \\ \frac{1}{36} & \text{for } z = 5 \\ 0 & \text{otherwise} \end{cases}$$

(e) Let A, B, and C be three events with probabilities given below.

(I) $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.1 + 0.1}{0.1 + 0.1 + 0.1 + 0.05} = \dfrac{4}{7}$

(II) $P(C \mid B) = \dfrac{P(C \cap B)}{P(B)} = \dfrac{0.1 + 0.05}{0.1 + 0.1 + 0.1 + 0.05} = \dfrac{3}{7}$

(III) $P(B \mid A \cup C) = \dfrac{P(B \cap (A \cup C))}{P(A \cup C)} = \dfrac{0.1 + 0.1 + 0.05}{0.1 + 0.1 + 0.05 + 0.2 + 0.1 + 0.15} =$
$\dfrac{5}{14}$

(IV) $P(B \mid A, C) = P(B \mid A \cap C) = \dfrac{P(B \cap (A \cap C))}{P(A \cup C)} = \dfrac{0.1}{0.1 + 0.1} = \dfrac{1}{2}$

(f) $\dfrac{\binom{5}{1}\binom{95}{2}}{\binom{100}{3}} = \dfrac{\left(\dfrac{5!}{1!(5-1)!}\right)\left(\dfrac{95!}{2!(95-2)!}\right)}{\left(\dfrac{100!}{3!(100-3)!}\right)} = \dfrac{5 \cdot 4465}{161700} = 13.81\%$