

Coin Probabilities

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Problem 24-2

Suppose we have a coin that lands on heads with probability k and tails with probability $1-k$. We flip the coin 5 times and get HHTTH.

a. Compute the likelihood of the observed outcome if the coin were fair (i.e. $k=0.5$). *SHOW YOUR WORK!*

$$P(\text{HHTTH} | k = 0.5) = P(H | k = 0.5) \cdot P(H | k = 0.5) \cdot P(T | k = 0.5) \cdot P(T | k = 0.5) \cdot P(H | k = 0.5) = ?$$

Check: your answer should come out to 0.03125.

b. Compute the likelihood of the observed outcome if the coin were slightly biased towards heads, say $k=0.55$. *SHOW YOUR WORK!*

$$P(\text{HHTTH} | k = 0.55) = P(H | k = 0.55) \cdot P(H | k = 0.55) \cdot P(T | k = 0.55) \cdot P(T | k = 0.55) \cdot P(H | k = 0.55) = ?$$

Check: your answer should round to 0.03369.

c. Compute the likelihood of the observed outcome for a general value of p . Your answer should be a function of k .

$$P(\text{HHTTH} | k) = P(H | k) \cdot P(H | k) \cdot P(T | k) \cdot P(T | k) \cdot P(H | k) = ?$$

Check: When you plug in $k = 0.5$, you should get the answer from part (a), and when you plug in $k = 0.55$, you should get the answer from part (b).

d. Plot a graph of $P(\text{HHTTH} | k)$ for $0 \leq k \leq 1$, and include the graph in your writeup.

Note: You can use the same plotting code as usual. You'll just need to come up with a list of many data points on the function $y = P(\text{HHTTH} | k)$.

Solution

a.

$$P(\text{H} | k = 0.5) = P(\text{T} | k = 0.5) = 0.5$$

$$P(\text{HHTTH} | k = 0.5) = 0.5^5 = 0.03125$$

b.

$$P(\text{H} | k = 0.55) = 0.55$$

$$P(\text{T} | k = 0.55) = 0.45$$

$$P(\text{HHTTH} | k = 0.55) = 0.55^3 \cdot 0.45^2 = 0.166375 \cdot 0.2025 = 0.033369 \text{ (rounded)}$$

c.

$$P(\text{H} | k) = k$$

$$P(\text{T} | k) = 1 - k$$

$$P(\text{HHTTH} | k) = k^3 \cdot (1 - k)^2$$

d.

