

Probability, Statistics, and Randomness

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Problem 30-1

Problems in form of images...

Solution

Problem 1:

a.

$$P(T \leq 1) = \frac{1}{16} = 0.0625$$

b.

$$P(2 \leq T < \infty) = P(T < \infty) - P(T \leq 2) = 1 - \frac{1}{4} = 0.75$$

c.

$$P(1 \leq T \leq 3) = P(T \leq 3) - P(T \leq 1) = \frac{9}{16} - \frac{1}{16} = 0.5$$

Problem 2:

$$P(3 \geq T \geq 2) = \frac{P(T \geq 2) - P(T \geq 3)}{P(T \geq 2)} = \frac{e^{-\frac{2}{5}} - e^{-\frac{3}{5}}}{e^{-\frac{2}{5}}} = 0.1813$$

Problem 3:

a.

$$\begin{aligned}\sum_{k=1}^{\infty} \frac{c}{3^k} &= c \left(\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots \right) = 1 \\ &= c \cdot \frac{1}{2} = 1 \\ &= c = 2\end{aligned}$$

b.

$$\begin{aligned}P(2, 4, 6) = P(2) + P(4) + P(6) &= \frac{2}{9} + \frac{2}{81} + \frac{2}{729} \\ &= \frac{162}{729} + \frac{18}{729} + \frac{2}{729} \\ &= \frac{182}{729} = 0.2497\end{aligned}$$

c.

$$\begin{aligned}\sum_{k=3}^{\infty} P(k) &= \sum_{k=3}^{\infty} \frac{2}{3^k} = 2 \left(\frac{1}{27} + \frac{1}{81} + \frac{1}{243} + \dots \right) \\ &= 2 \cdot \frac{1}{18} \\ &= \frac{1}{9} = 0.1111\end{aligned}$$