Logistic Regression

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November 19, 2020

Problem 33-1

Suppose you are again given the following dataset:
$data = [(1, 0.2), (2, 0.25), (3, 0.5)]$
Fit a logistic regression model $y = \dfrac{1}{1 + e^{a+bx}}$ *by hand.*

1. *Re-express the model in the form* $a + bx =$ *some function of* y *(i.e. isolate* $a+bx$ *in the logistic regression model). Hint: your function of* y *will involve ln.*

2. *Set up a system of equations and turn the system into a matrix equation.*

3. *Find the best approximation to the solution of that matrix equation by using the pseudoinverse.*

4. *Substituting your solution for the coefficients of the model, and plot the model along with the 3 given data points on the same graph to ensure that the model fits the data points well.*

Solution

Step 1:

$$\begin{aligned}
\frac{1}{1 + e^{a+bx}} &= y \\
1 + e^{a+bx} &= \frac{1}{y} \\
e^{a+bx} &= \frac{1}{y} - 1 \\
a + bx &= ln\left(\frac{1}{y} - 1\right)
\end{aligned}$$

Step 2:

Plugging in all the given values, we get...

$ln(4) = a + (1)b$

$ln(3) = a + 2b$

$ln(1) = a + 3b$

We turn this into a matrix equation...

$$\begin{bmatrix} ln(4) \\ ln(3) \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix}$$

Step 3:

$$\begin{aligned} \begin{bmatrix} ln(4) \\ ln(3) \\ 0 \end{bmatrix} &= \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} \\ \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} ln(4) \\ ln(3) \\ 0 \end{bmatrix} &= \begin{bmatrix} 3 & 6 \\ 6 & 14 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} \\ \begin{bmatrix} 3 & 6 \\ 6 & 14 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} ln(4) \\ ln(3) \\ 0 \end{bmatrix} &= \begin{bmatrix} a \\ b \end{bmatrix} \\ \begin{bmatrix} 3 & 6 \\ 6 & 14 \end{bmatrix}^{-1} \begin{bmatrix} ln(4) + ln(3) \\ ln(4) + 2ln(3) \end{bmatrix} &= \begin{bmatrix} a \\ b \end{bmatrix} \\ \begin{bmatrix} 2.2146 \\ -0.69315 \end{bmatrix} &= \begin{bmatrix} a \\ b \end{bmatrix} \end{aligned}$$

Step 4: $y = 2.2146 - 0.69315x$: