Probability

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Problem 42-2

Problem given via images...

Solution

a.

a. $P(A \cap B) = P(A) + P(B) - P(A \cup B) = \frac{1}{2} + \frac{2}{3} - \frac{5}{6} = \frac{1}{3}$
b. No, A and B intersect
c. $P(C - (A \cup B))$ is just everything that's in C that isn't in A or B. We know that $A \cup B = \frac{5}{6}$, so $P(C - (A \cup B)) = 1 - \frac{5}{6} = \frac{1}{6}$
d. $P(C) = P(C - (A \cup B)) + P(C \cap (A \cup B)) = \frac{1}{6} + \frac{5}{12} = \frac{7}{12}$

b.

$$\begin{aligned}
\text{Var}[2X - Y] &= \text{Var}[2X] + \text{Var}[-Y] + 2\text{Cov}[2X, -Y] = 6 \\
&= 4\text{Var}[X] + \text{Var}[Y] + 2\text{E}[-2XY] - 2\text{E}[2X]\text{E}[-Y] = 6 \\
&= 4\text{Var}[X] + \text{Var}[Y] - 4\text{E}[XY] + 4\text{E}[X]\text{E}[Y] = 6 \\
&= 4\text{Var}[X] + \text{Var}[Y] - 4\text{Cov}[X, Y] = 6 \\
&= 4\text{Var}[X] + \text{Var}[Y] = 6
\end{aligned}$$

$$\begin{aligned}
\text{Var}[X + 2Y] &= \text{Var}[X] + \text{Var}[2Y] + 2\text{Cov}[X, 2Y] = 9 \\
&= \text{Var}[X] + 4\text{Var}[Y] + 2\text{E}[2XY] - 2E[X]E[2Y] = 9 \\
&= \text{Var}[X] + 4\text{Var}[Y] + 4\text{E}[XY] - 4E[X]E[Y] = 9 \\
&= \text{Var}[X] + 4\text{Var}[Y] + 4\text{Cov}[X, Y] = 9 \\
&= \text{Var}[X] + 4\text{Var}[Y] = 9
\end{aligned}$$

(Using second equation)... $\text{Var}[X] = 9 - 4\text{Var}[Y]$

(Using first equation)...

$$\begin{aligned} 36 - 15\text{Var}[Y] &= 6 \\ \text{Var}[Y] &= 2 \end{aligned}$$

Using $\text{Var}[Y] = 2$ we get that $\text{Var}[X] = 1$

c.

a. $X \in (0, 1, 2)$

b. $P(X \geq 1.5) = P(2) = \dfrac{1}{6}$

c. $P(0 < X < 2) = P(1) = \dfrac{1}{3}$

d. $P(X = 0|X < 2) = \dfrac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{3}} = \dfrac{1}{\frac{5}{3}} = \dfrac{3}{5}$

d.

$$P(z) = \begin{cases} \frac{1}{36},\ z = -5 \\ \frac{1}{18},\ z = -4 \\ \frac{1}{12},\ z = -3 \\ \frac{1}{9},\ z = -2 \\ \frac{5}{36},\ z = -1 \\ \frac{1}{6},\ z = 0 \\ \frac{5}{36},\ z = 1 \\ \frac{1}{9},\ z = 2 \\ \frac{1}{12},\ z = 3 \\ \frac{1}{18},\ z = 4 \\ \frac{1}{36},\ z = 5 \end{cases}$$

e.

a. $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.2}{0.35} = 0.571$

b. $P(C|B) = \dfrac{P(C \cap B)}{P(B)} = \dfrac{0.15}{0.35} = 0.429$

c. $P(B|A \cup C) = \dfrac{P(B \cap (A \cup C))}{P(A \cup C)} = \dfrac{0.25}{0.7} = 0.357$

d. $P(B|A \cap C) = \dfrac{P(B \cap (A \cap C))}{P(A \cap C)} = \dfrac{0.1}{0.2} = 0.5$

f.

$$P(\text{one defective product}) = \frac{5}{100} \cdot \frac{95}{99} \cdot \frac{93}{98} \cdot 3 = 15\% \text{ chance}$$