Machine Learning Assignment 20

1

A: Using integration, show that this is a valid distribution, i.e. all the probability integrates to 1.

$$\int_{-\infty}^{\infty} p(x)dx =$$

$$\int_0^{\infty} p(x)dx + \int_{-\infty}^{0} p(x)dx =$$

$$\int_0^{\infty} 2e^{-2x}dx + \int_{-\infty}^{0} 0dx =$$

$$\text{-e}^{-2x}|_{x=0}^{x=\infty} + 0$$

$$\left(\frac{1}{-e^{2\infty}}\right) - (-e^0) =$$

$$\text{-(-1)} = 1$$

B: Given that X $\sim$p$_2$, compute P(0 ¡ X $\leq$ 1).

$$\int_0^1 p(x)dx =$$

$$\int_0^1 2e^{-2x}dx =$$

$$\text{-e}^{-2x}|_{x=0}^{x=1} =$$

$$\text{-e}^{-2} - (-1) = 1 - e^{-2}$$

C: Given that X $\sim$ p$_2$, compute $\boldsymbol{E[X]}$

$$\text{E[X]} =$$

$$\int_{-\infty}^{\infty} xp(x)dx =$$

$$\int_{0}^{\infty} xp(x)dx + \int_{-\infty}^{0} xp(x)dx =$$

$$\int_{0}^{\infty} x \cdot 2e^{-2x}dx + \int_{-\infty}^{0} x \cdot 0dx =$$

$$\left(-\frac{2xe^{-2x}+e^{-2x}}{2}\right)|_{x=0}^{x=\infty} + 0$$

$$\left(-\frac{2(\infty)e^{-2\infty}+e^{-2\infty}}{2}\right) - \left(-\frac{2(0)e^{0}+e^{0}}{2}\right) =$$

$$0 + \frac{1}{2} = \frac{1}{2}$$

D: Given that X $\sim$ p$_2$, compute Var$[\boldsymbol{X}]$

$$\text{Var[X]} = \text{E[(X - E[N]}^2] = E[(X - \tfrac{1}{2})^2]$$

$$\int_{-\infty}^{\infty}(x - \tfrac{1}{2})^2 p(x)dx =$$

$$\int_{0}^{\infty}(x - \tfrac{1}{2})^2 p(x)dx + \int_{-\infty}^{0}(x - \tfrac{1}{2})^2 p(x)dx =$$

$$\int_{0}^{\infty}(x - \tfrac{1}{2})^2 \cdot 2e^{-2x}dx + \int_{-\infty}^{0}(x - \tfrac{1}{2})^2 \cdot 0dx =$$

$$\int_{0}^{\infty} 2xe^{-2x} + 2x^2e^{-2x}dx + \int_{-\infty}^{0} 0dx =$$

$$\left(-\frac{4x^2e^{-2x}+e^{-2x}}{4}\right) + 0|_{x=0}^{x=\infty}$$

$$\left(-\frac{4(\infty)^2e^{-2(\infty)}+e^{-2(\infty)}}{4}\right) - \left(-\frac{4(0)^2e^{0}+e^{0}}{4}\right) =$$

$$0 + \frac{1}{4} = \frac{1}{4}$$