

Machine Learning Assignment 21

A

$$\begin{aligned}\int_{-\infty}^{\infty} p(x) dx &= \int_{-\infty}^3 p(x) dx + \int_7^3 p(x) dx + \int_7^{\infty} p(x) dx \\ &= \int_{-\infty}^3 0 dx + \int_7^3 k dx + \int_7^{\infty} 0 dx \\ &= [kx]_{x=3}^{x=7} \\ &= 7k - 3k \\ &= 4k \\ k &= \frac{1}{4}\end{aligned}$$

B

$$\begin{aligned}E[X] &= \int_{-\infty}^{\infty} x \cdot p(x) dx \\ &= \int_{-\infty}^3 x \cdot p(x) dx + \int_3^7 x \cdot p(x) dx + \int_7^{\infty} x \cdot p(x) dx \\ &= \int_{-\infty}^3 x \cdot (0) dx + \int_3^7 x \cdot \frac{1}{4} dx + \int_7^{\infty} x \cdot (0) dx \\ &= \left[\frac{x^2}{8} \right]_{x=3}^{x=7} \\ &= \frac{7^2}{8} - \frac{3^2}{8} \\ &= \frac{49}{8} - \frac{9}{8} \\ &= \frac{40}{8} \\ &= 5\end{aligned}$$

C

$$\begin{aligned} \text{Var}[X] &= E[(X - E[X])^2] \\ &= E[(x - 5)^2] \\ &= \int_{-\infty}^{\infty} (x - 5)^2 \cdot p(x) \, dx \\ &= \int_{-\infty}^3 (x - 5)^2 \cdot p(x) \, dx + \int_3^7 (x - 5)^2 \cdot p(x) \, dx + \int_7^{\infty} (x - 5)^2 \cdot p(x) \, dx \\ &= \int_{-\infty}^3 (x - 5)^2 \cdot (0) \, dx + \int_3^7 (x - 5)^2 \cdot \frac{1}{4} \, dx + \int_7^{\infty} (x - 5)^2 \cdot (0) \, dx \\ &= \left[\frac{1}{4} \cdot \left(\frac{(x - 5)^3}{3} \right) \right]_{x=3}^{x=7} \\ &= \frac{1}{4} \cdot \left(\frac{(7 - 5)^3}{3} - \frac{(3 - 5)^3}{3} \right) \\ &= \frac{1}{4} \cdot \left(\frac{8}{3} + \frac{8}{3} \right) \\ &= \frac{1}{4} \cdot \frac{8}{3} \\ &= \frac{4}{3} \end{aligned}$$