Machine Learning Assignment 24

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Problem 1

A coin is flipped and gets *HHTTH*

A: Compute the likelihood of the observed outcome if the coin were fair (i.e. $k = 0.5$).

$$\begin{aligned} P(\text{HHTTH} \mid k = 0.5) &= P(\text{H} \mid k = 0.5) \cdot P(\text{H} \mid k = 0.5) \cdot P(\text{T} \mid k = 0.5) \cdot P(\text{T} \mid k = 0.5) \cdot P(\text{H} \mid k = 0.5) \\ &= 0.5 \cdot 0.5 \cdot (1 - 0.5) \cdot (1 - 0.5) \cdot 0.5 \\ &= 0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 \\ &= 0.5^5 \\ &= 0.03125 \end{aligned}$$

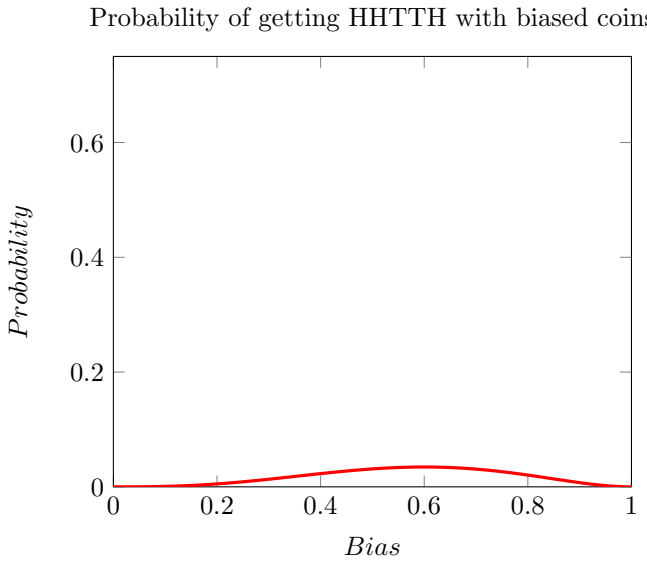
B: Compute the likelihood of the observed outcome if the coin were slightly biased towards heads, say $k = 0.55$.

$$\begin{aligned} P(\text{HHTTH} \mid k = 0.55) &= P(\text{H} \mid k = 0.55) \cdot P(\text{H} \mid k = 0.55) \cdot P(\text{T} \mid k = 0.55) \cdot P(\text{T} \mid k = 0.55) \cdot P(\text{H} \mid k = 0.55) \\ &= 0.55 \cdot 0.55 \cdot (1 - 0.55) \cdot (1 - 0.55) \cdot 0.55 \\ &= 0.55 \cdot 0.55 \cdot 0.45 \cdot 0.45 \cdot 0.55 \\ &= 0.55^3 \cdot 0.45^2 \\ &= 0.03369 \end{aligned}$$

C: Compute the likelihood of the observed outcome for a general value of p. Your answer should be a function of k.

$$\begin{aligned} P(\text{HHTTH} \mid k) &= P(\text{H} \mid k) \cdot P(\text{H} \mid k) \cdot P(\text{T} \mid k) \cdot P(\text{T} \mid k) \cdot P(\text{H} \mid k) \\ &= k \cdot k \cdot (1 - k) \cdot (1 - k) \cdot k \\ &= k^3 \cdot (1 - k)^2 \\ &= k^3 \cdot (k^2 - 2k + 1) \\ &= k^5 - 2k^4 + k^3 \end{aligned}$$

D:


Probability of getting HHTTH with biased coins
Probability
0
0.2
0.4
0.6
0
0.2
0.4
0.6
0.8
1
Bias