Machine Learning Assignment 26

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Part 1

Suppose that you take a bus to work every day. Bus A arrives at 8am but is x minutes late with $x \sim U(0, 20)$. Bus B arrives at 8:10 but with $x \sim U(0, 10)$. The bus ride is 20 minutes and you need to arrive at work by 8:30.

A: If you take bus A, what time do you expect to arrive at work?

Knowing that the Expected Value is $\mathrm{E}[X] = \dfrac{a+b}{2}$, then it is easy to find the expected time to arrive. As the number of minuets the bus runs late is the uniform distribution from (0, 20). Replacing a for zero and 20 for b, the answer is that the bus will likely be 10 minuets late, meaning you will arrive around 8:30

B: If you take bus B, what time do you expect to arrive at work?

Going off of the same logic as Part A, except with the number of minutes the bus runs late being the uniform distribution from (0, 10). The bus will likely be 5 minutes late, meaning you will arrive around 8:35

C: If you take bus A, what is the probability that you will arrive on time to work?

As this is a uniform probability distribution, there is the same chance of the bus arriving at any time. As the bus can be at most 10 minuets late to arrive at work on time, you have... a 50% chance of getting to work on time

$$P(X \leq 10) = \int_0^{10} \frac{1}{20}\,\mathrm{d}x$$
$$* = \frac{1}{2} = 0.5$$

D: If you take bus B, what is the probability that you will arrive on time to work?

Once again, following the logic above will lead to the same conclusion. However, here the bus MUST be on time in order to get to work on time. As the bus cannot be early you have a... 9% chance of getting to work on time.

$$\begin{aligned} P(X = 0) &= \int_0^0 \frac{1}{10} \\ &= 0 \end{aligned}$$

Part 2

Continuing the scenario above, there is a third option that you can use to get to work: you can jump into a wormhole and (usually) come out almost instantly at the other side. The only issue is that time runs differently inside the wormhole, and while you're probably going to arrive at the other end very quickly, there's a small chance that you could get stuck in there for a really long time.
The number of seconds it takes you to come out the other end of the wormhole follows an exponential distribution $\text{Exp}(\lambda = 4)$.

A: How long do you expect it to take you to come out of the wormhole?

Knowing that the Expected Value of an exponential distribution is $\text{E}[X] = \frac{1}{\lambda}$, then it is easy to find the expected time to arrive. As $\lambda = 4$ in this distribution, you just plug the 4 into the equation to get that

B: What's the probability of taking longer than a second to come out of the wormhole?

To find the probability of that, you just have to take the integral of the exponential distribution from zero to infinity. This leaves you with around... a 1.8% chance of being in the wormhole for over a second.

$$\begin{aligned} P(X > 1) &= \int_1^\infty 4 \cdot e^{-4x} \, \mathrm{d}x \\ &= \left[-e^{-4x}\right]_1^\infty \\ &= -e^{-4\infty} - \left(-e^{-4}\right) \\ &= e^{-4} \\ &\approx 0.018 \end{aligned}$$

C: Fill in the blank: the probability of coming out of the wormhole within ___ seconds is 99.999%

Around 2.878 seconds

$$
\begin{aligned}
0.99999 &= \int_0^u 4e^{-4t} \\
&= \left[-e^{-4t}\right]_0^u \\
&= 1 - e^{-4t}
\end{aligned}
$$

$$
\begin{aligned}
0.00001 &= e^{-4t} \\
ln(0.00001) &= -4t \\
t &= -\frac{ln(0.00001)}{4} \\
&\approx 2.878231
\end{aligned}
$$

D: Your friend says that you shouldn't use the wormhole because there's always a chance that you might get stuck in it for over a day, and if you use the wormhole often, then that'll probably happen sometime within your lifetime. Is this a reasonable fear? Why or why not? Justify your answer by computing the probability that you'll get stuck in the wormhole for over a day if you use the wormhole 10 times each day for 80 years.

Oh, this is not a reasonable fear at all. When I put in the chances of getting for a day, 86400 seconds, my calculator spat out zero. So the chances of getting stuck in there for a day is... 0%

$$
\begin{aligned}
P(X > 86400) &= \\
\int_{86400}^{\infty} 4 \cdot e^{-4x}\,\mathrm{d}x &= \\
\left[-e^{-4x}\right]_{86400}^{\infty} &= \quad \text{(IBP)} \\
-e^{-4\infty} - \left(-e^{-4 \cdot 86400}\right) &= \\
e^{-345600} &= 0
\end{aligned}
$$