Machine Learning Assignment 33

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Problem 33-1

To start off you have to take the original equation and set it to the $ax+b$ format.

$$\begin{aligned} y &= \frac{1}{1+e^{ax+b}} \\ (y)^{-1} &= 1+e^{ax+b} \\ (y)^{-1}-1 &= e^{ax+b} \\ ln((y)^{-1}-1) &= ax+b \end{aligned}$$

Now that we have the equation we can find a and b.

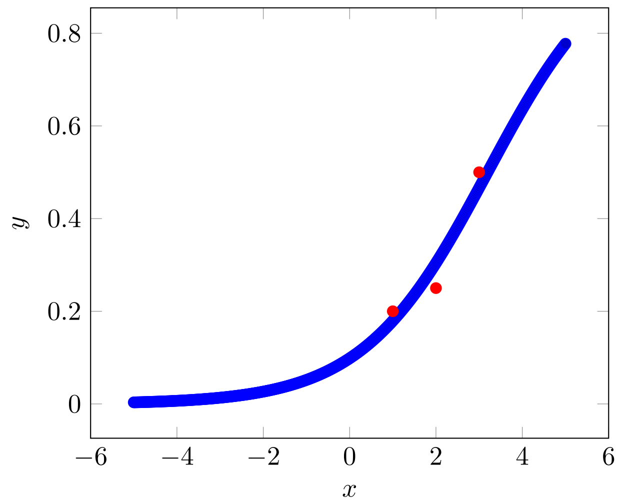
$$\begin{bmatrix} ln((0.2)^{-1}-1) \\ ln((0.25)^{-1}-1) \\ ln((0.5)^{-1}-1) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} ln((0.2)^{-1}-1) \\ ln((0.25)^{-1}-1) \\ ln((0.5)^{-1}-1) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} ln((0.2)^{-1}-1) \\ ln((0.25)^{-1}-1) \\ ln((0.5)^{-1}-1) \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 6 & 14 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\begin{bmatrix} 3 & 6 \\ 6 & 14 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} ln((0.2)^{-1}-1) \\ ln((0.25)^{-1}-1) \\ ln((0.5)^{-1}-1) \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\begin{bmatrix} -0.69314\ldots \\ 2.21459\ldots \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix} \quad \text{(Wolfram Alpha)}$$


0.8
0.6
0.4
0.2
0
y
−6
−4
−2
0
2
4
6
x

Problem 33-2

A: Given that X $\sim$ p(x), where p(x) is a continuous distribution, prove that for any real number a we have E[aX] = aE[X].

$$\begin{aligned} E[aX] &= \int_{-\infty}^{\infty} a \cdot x \cdot p(x) \, \mathrm{d}x \\ &= a \cdot \int_{-\infty}^{\infty} x \cdot p(x) \, \mathrm{d}x \\ &= aE[X] \end{aligned}$$

B: Given that X $\sim$ p(x) where p(x) is a continuous probability distribution, prove the identity Var[X] = E[X^2] - E[X]^2.

$$\begin{aligned} Var[X] &= \int_{-\infty}^{\infty} (X - E[X])^2 \cdot p(x) \, \mathrm{d}x \\ &= \int_{-\infty}^{\infty} (x^2 - 2xE[X] + E[X]^2) \cdot p(x) \, \mathrm{d}x \\ &= \int_{-\infty}^{\infty} x^2 \cdot p(x) - \int_{-\infty}^{\infty} (2xE[X] - E[X]^2) \cdot p(x) \\ &= E[X^2] - E[2xE[X] - E[X]^2] \\ &= E[X^2] - (2E[X] \cdot E[X] - E[X]^2) \\ &= E[X^2] - E[X]^2 \end{aligned}$$