Machine Learning Assignment 42

Maia Dimas

Section A

A: Find $P(A \cap B)$

$$
\begin{aligned}
P(A \cap B) &= P(A) + P(B) - P(A \cup B) \\
&= \frac{1}{2} + \frac{2}{3} - \frac{5}{6} \\
&= \frac{1}{3}
\end{aligned}
$$

B: Do A, B, and C form a partition of S

No, as there is overlap between A and B. P(A ∩ B) is not zero, meaning that these sets are not disjointed.

C: Find $P(C$ - $(A \cup$ B$))$

$$P(C - (A \cup B)) = P(C) - P(C \cap (A \cup B))$$

D: If P$(C \cap (A \cup B)) = \frac{5}{12}$, find $P(C)$

$$
\begin{aligned}
P(C \cap (A \cup B)) &= P(C) + P(A \cup B) - P(C \cup A \cup B) \\
&= P(C) + \frac{5}{6} - 1
\end{aligned}
$$

$$
\begin{aligned}
\frac{5}{12} &= P(C) - \frac{1}{6} \\
P(C) &= \frac{7}{12}
\end{aligned}
$$

Section B

$$
\begin{aligned}
6 &= Var[2X - Y] \\
&= Var[2X] + Var[-Y] + 2Cov[2X, -Y] \\
&= E[(2X - 2E[X])^2] + E[(-Y - E[-Y])^2] + 2Cov[2x, -Y] \\
&= E[4(X - E[X])^2] + E[(Y - E[Y])^2] - 4E[XY] + 4E[X]E[Y] \\
&= 4Var[X] + Var[Y]
\end{aligned}
$$

$$
\begin{aligned}
9 &= Var[X + 2Y] \\
&= Var[X] + Var[2Y] + 2Cov[X, 2Y] \\
&= Var[X] + 4Var[Y]
\end{aligned}
$$

$$
\begin{aligned}
6 &= 4Var[X] + Var[Y] \\
9 &= Var[X] + 4Var[Y]
\end{aligned}
$$

$$
\begin{aligned}
1 &= Var[X] \\
2 &= Var[Y]
\end{aligned}
$$

Section C

A: Find R_X, the range of the random variable X

$$R_X = \{0, 1, 2\}$$

B: Find $\boldsymbol{P(X \geq 1.5)}$

$$
\begin{aligned}
P(X \geq 1.5) &= 1 - P(X < 1.5) \\
&= 1 - (P(X \leq 1.5) - P(X = 1.5)) \\
&= 1 - \left(\left(\frac{1}{2} + \frac{1}{3}\right) - 0\right) \\
&= \frac{1}{6}
\end{aligned}
$$

C: Find $\boldsymbol{P(0 < X < 2)}$

$$
\begin{aligned}
P(0 < X < 2) &= P(1) \\
&= \frac{1}{3}
\end{aligned}
$$

D: Find $P(X = 0 \mid X < 2)$

$$
\begin{aligned}
P(X=0|X<2) &= \frac{P(X=0)\cdot P(X<2|X=0)}{P(X<2)} \\
&= \frac{\frac{1}{2}\cdot 1}{\frac{5}{6}} \\
&= \frac{3}{5}
\end{aligned}
$$

Section D

$$
P(z) = \begin{cases}
\frac{1}{6}, & z = 0 \\
\frac{5}{36}, & z = \pm 1 \\
\frac{1}{9}, & z = \pm 2 \\
\frac{1}{12}, & z = \pm 3 \\
\frac{1}{18}, & z = \pm 4 \\
\frac{1}{36}, & z = \pm 5
\end{cases}
$$

Section E

A: Find $P(A \mid B)$

Around 57%

$$
\begin{aligned}
P(A|B) &= \frac{P(A\cap B)}{P(B)} \\
&= \frac{0.2}{0.35} \\
&= 0.57143
\end{aligned}
$$

B: Find $P(C \mid B)$

Around 43%

$$
\begin{aligned}
P(C|B) &= \frac{P(C\cap B)}{P(B)} \\
&= \frac{0.15}{0.35} \\
&= 0.42857
\end{aligned}
$$

C: Find $P(B \mid A \cup C)$

$$
\begin{aligned}
P(B|A \cup C) &= \frac{P(B \cap (A \cup C))}{P(A \cup C)} \\
&= \frac{0.15}{0.5} \\
&= 0.3
\end{aligned}
$$

D: Find $P(B \mid A,\ C) = P(B \mid A \cap \mathrm{C})$

$$
\begin{aligned}
P(B|A \cap C) &= \frac{P(B \cap (A \cap C)}{P(B)} \\
&= 0.5
\end{aligned}
$$

$$
\begin{aligned}
P(B|A, C) &= \frac{P(B \cap A, C)}{P(A, C)} \\
&= \frac{0.1}{0.2} \\
&= 0.5 \\
\text{So thus } P(B|A, C) &= P(B|A \cap C)
\end{aligned}
$$

Section F

Around 14%

$$
\begin{aligned}
P(1\, defective) &= 3\left(\frac{95}{100} \cdot \frac{94}{99} \cdot \frac{5}{98}\right) \\
&= 0.13806
\end{aligned}
$$