Assignment 19

Nathan Allen

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1 PART 1

(a) *a. Write the probability distribution p4(n) for getting n heads on 4 coin flips, where the coin is a fair coin (i.e. it lands on heads with probability 0.5).*

Answer We use the formula $4!/n!(4-n)!$ for every amount of heads to get the instances of heads. We then divide by 2^4 or 16 or $4!/n!(4-n)! * (1/16)$ to get [.0675, .25, .375, .25, .0675]
(a) *b. Let N be the number of heads in 4 coin flips. Then* $Np4$*. Intuitively, what is the expected value of N? Explain the reasoning behind your intuition.*

Answer Intuitively we would think that there would be 2 heads. This is because a coin flip is supposed to have fifty-fifty odds so there would be an equal amount of heads and tails.
(a) *c. Compute the expected value of N, using the definition*

$$E[N] = np(n)$$

. The answer you get should match your answer from (b).

Answer

$n*p(n) = (0*1/16)+(1*1/4)+(2*6/16)+(3*1/4)+(4*1/16) = 1/4+3/4+3/4+1/4 = 2$

This matches with what I got in b.
(a) *d. Compute the variance of N, using the definition*

$$Var[N] = E[(NE[N])^2]$$

.Your answer should come out to 1.

Answer So we know from part c that E[N] = 2 This breaks down the the equation to $E[(N-2)^2]$

2 PART 2

(a) *a. Write the probability distribution p4,k(n) for getting n heads on 4 coin flips, where the coin is a biased coin that lands on heads with probability k. If you substitute k=0.5, you should get the same result that you did in part 1a.*

Answer Lets say the probability of getting a head is k.

$$k(n) = 4!/n!(4-n)! * ((k^n) * ((1-k)^(4-n))$$

if you plug in .5 for k you would get

$$k(n) = 4!/n!(4-n)!*((.5^n)*((.5)^(4-n))) = 4!/n!(4-n)!*(.5^4) = 4!/n!(4-n)!*(1/16)$$

if you write it out you get $((k^n) * ((1-k)^(4-n))) * [1, 4, 6, 4, 1]$ plug in .5 you get $1/16 * [1, 4, 6, 4, 1] = [.0675, .25, .375, .25, .0675]$

(a) *b. Let N be the number of heads in 4 coin flips of a biased coin. Then Np4,k. Intuitively, what is the expected value of N? Your answer should be in terms of k. Explain the reasoning behind your intuition. If you substitute k=0.5, you should get the same result that you did in part 1b.*

Answer You should get a value that is the number of flips times k. If you plug in .5 you get 2 which is the same a 1-b

(a) *c. Compute the expected value of N, using the definition* $E[N] = np(n)$*. The answer you get should match your answer from (b*

Answer $E[N] = np(n)$ in the context of k become equal to $(0*1*((k^n)*((1-k)^(4-n)))) + (1*4*((k^n)*((1-k)^(4-n)))) + (2*6*((k^n)*((1-k)^(4-n)))) + (3*4*((k^n)*((1-k)^(4-n)))) + (4*1*((k^n)*((1-k)^(4-n))))$ if I plug in .5 you get $(0*1/16)+(1*1/4)+(2*6/16)+(3*1/4)+(4*1/16) = 1/4+3/4+3/4+1/4 = 2$