# Assignment 21

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**(a)** *Find the value of k such that p(x) is a valid probability distribution.*

**Answer** The probability distribution must integrate to 1 so

$$1 = \int_3^7 k\mathrm{d}x$$

$$= \ k * x|_3^7$$

$$= 7k - 3k$$

$$= 4k = 1$$

$$k = 1/4$$

**(b)** *Given that $X \approx U[3,7]$, compute $E[X]$.*

**Answer**

$$E[X] = \int_{-\infty}^{\infty} xp(x)\mathrm{d}x$$

Since $p(x) = 0$ when $x \notin [3,7]$

$$E[X] = \int_3^7 x * \frac{1}{4}\mathrm{d}x$$

$$= \left.\frac{x^2}{8}\right|_3^7$$

$$= \frac{49}{8} - \frac{9}{8}$$

$$= \frac{40}{8} = 5$$

**(c)** *Given that $X \approx U[3,7]$, compute Var[X].*

**Answer** since $\mathrm{Var}[N] = E[(NE[N])^2]$ and $E[N] = 5$ $\mathrm{Var}[N] = E[(N-5)^2]$

$$= \frac{1}{4} * \int_3^7 x^2 - 10x + 25\mathrm{d}x$$

$$= \frac{1}{4} * \frac{x^3}{3} - 5 * x^2 + 25x \Big|_3^7$$
$$= \frac{1}{4} * (\frac{133}{3} - 39)$$
$$= \frac{1}{4} * (\frac{16}{3})$$
$$= \frac{4}{3}$$