Assignment 24

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1 Part 1

2 Suppose that you take a bus to work every day. Bus A arrives at 8am but is x minutes late with $x \sim U(0, 20)$. Bus B arrives at 8:10 but with $x(0, 10)$. The bus ride is 20 minutes and you need to arrive at work by 8:30. Remember that U(a,b) means the uniform distribution on [a,b]. See problem 23-2 if you need a refresher on exponential distributions. Recall the formulas for the mean and variance of uniform distributions: If $X(a, b)$, then $E[X] = \frac{a+b}{2}$ and $Var(X) = \frac{(ba)^2}{12}$. You can use these formulas without any further justification.

(a) *If you take bus A, what time do you expect to arrive at work? Justify your answer.*

Answer 8:30 since the bus ride is 20 minutes and the E[X] where $x \sim U(0, 20)$ is $\frac{20+0}{2} = 10$ 20 + 10 + 8:00 = 8:30

(b) *. If you take bus B, what time do you expect to arrive at work? Justify your answer.*

Answer 8:35 since the bus ride is 20 minutes and the E[X] where $x \sim U(0, 10)$ is $\frac{10+0}{2} = 5$ 20 + 5 + 8:10 = 8:35

(c) *If you take bus A, what is the probability that you will arrive on time to work? Justify your answer*

Answer 1/2 since the expected value is on time for work and therefore the midpoint for variance.

(d) *If you take bus B, what is the probability that you will arrive on time to work? Justify your answer*

Answer

3 Part 2

4 Continuing the scenario above, there is a third option that you can use to get to work: you can jump into a wormhole and (usually) come out almost instantly at the other side. The only issue is that time runs differently inside the wormhole, and while you're probably going to arrive at the other end very quickly, there's a small chance that you could get stuck in there for a really long time. The number of seconds it takes you to come out the other end of the wormhole follows an exponential distribution $Exp(= 4)$. See problem 23-2 if you need a refresher on exponential distributions. Recall the formulas for the mean and variance of exponential distributions: If $X \sim Exp()$, then $E[X] = \frac{1}{\lambda}$ and $Var(X) = \frac{1}{\lambda^2}$. You can use these formulas without any further justification.

(a) *How long do you expect it to take you to come out of the wormhole? Justify your answer.*

Answer .25 since $\lambda == 4$ and $E[X] = \frac{1}{\lambda}$ so $\frac{1}{4} = .25$

(b) *What's the probability of taking longer than a second to come out of the wormhole? Justify your answer.*

Answer 0 since the variance is 1/16 plus the expected value 1/4 gets you 5/16 which is not greater than 1.

(c) *Fill in the blank: the probability of coming out of the wormhole within* $_{seconds is 99.999 percent. Justify your answer}$

Answer *5/16 since the variance is 1/16 plus the expected value 1/4 gets you 5/16. This is at the very end of the spectrum so the odds are really low of getting this.*

(d) Your friend says that you shouldn't use the wormhole because there's always a chance that you might get stuck in it for over a day, and if you use the wormhole often, then that'll probably happen sometime within your lifetime. Is this a reasonable fear? Why or why not? Justify your answer by computing the probability that you'll get stuck in the wormhole for over a day if you use the wormhole 10 times each day for 80 years.

Answer