1 Suppose you toss a coin 10 times and get the result HHHHT HHHHH. From this result, you estimate that the coin is biased and generally lands on heads 90 percent of the time. But how sure can you be? Let's quantify it.

(a) *Compute the likelihood P(HHHHT HHHHH—k) where P(H)=k. Remember that the likelihood is just the probability of getting the result HHHHT HHHHH under the assumption that P(H)=k. Your answer should be expressed in terms of k.*

Answer $P(HHHHTHHHHH|k) = P(H) * P(H) * P(H) * P(H) * P(T) * P(H) * P(H) * P(H) * P(H) * P(H) = k^9 - k^{10})$

(b) *The likelihood P(HHHHT HHHHH—k) can almost be interpreted as a probability distribution for k. The only problem is that it doesn't integrate to 1. Create a probability distribution P(k—HHHHT HHHHH) that is proportional to the likelihood P(HHHHT HHHHH—k). In other words, find the function P(k) such that* $P(k) = cP(HHHHTHHHHH|k)$ *for some constant c, and*

$$\int_0^1 P(k|HHHHTHHHHH)\,dk = 1.$$

Answer $\int_0^1 c * P(k|HHHHTHHHHH)\,dk = c * \int_0^1 k^9 - k^{10}\,dk$

$$= c(\frac{k^{10}}{10} - \frac{k^{11}}{11})\Big|_0^1$$
$$= \frac{c}{10} - \frac{c}{11} = 1$$
$$c = 110$$

(c) *Using the prior distribution P(k)U[0,1], what was the prior probability that the coin was biased towards heads? In other words, what was P(k¿0.5)?*

Answer $P(k) = \frac{1}{1-0} = 1$

$$\int_{0.5}^1 1\,dk = 0.5$$

(d) *Using the posterior distribution P(k—HHHHT HHHHH), what was the posterior probability that the coin was biased towards heads? In other words, what is P(k¿0.5—HHHHT HHHHH)?*

Answer $\int_{0.5}^1 c * P(k|HHHHTHHHHH)\,dk = 110 * \int_{0.5}^1 k^9 - k^{10}\,dk = 0.994141$

(e) *Compare your answers in parts (c) and (d). Did the probability that the coin was biased towards heads increase or decrease, after observing the sequence of flips? Why does this make intuitive sense?*

Answer The biased increased by 444141. This makes sense since a coin that only has one tails out of ten flips is highly unlikely.

(f) *Using the posterior distribution, what is the most probable value of k? In other words, what is value of k at which P(k—HHHHT HHHHH) reaches a maximum? Show your work using the first or second derivative test*

Answer $P(k|HHHHTHHHHH) = 110 * (k^9 - k^{10})$

$$P'(k|HHHHTHHHHH) = 110(9k^8 - 10k^9)$$
$$110 * (9k^8 - 10k^9) = 0$$
$$110k^8(9 - 10k) = 0$$
$$k = 0, 9/10$$
$$P''(k|HHHHTHHHHH) = 110(72k^7 - 90k^8)$$
$$P''(0|HHHHTHHHHH) = 110(72 * 0^7 - 90 * 0^8) = 0$$
$$P''(1|HHHHTHHHHH) = 110(72 * 9/10^7 - 90 * 9/10^8) = \frac{-473513931}{1000000}$$

Since it is negative $\frac{9}{10}$ ii a max

(g) *Why does your answer to (f) make sense? What's the intuition here?*

Answer Since the coin landed 9/10 heads this result makes sense since we already have the actual result.

(h) *What is the probability that the bias k lies within 0.05 of your answer to part (g)? In other words, what is the probability that 0.85¡k¡0.95?*

Answer $P(0.85 < k < 0.95|HHHHTHHHHH) = 110 * \int_{0.85}^{0.95} k^9 - k^{10}\, dk = 0.405919$

(i) *Fill in the blank: you can be 99 percent sure that P(H) is at least* .

Answer $.99 = P(k > q) = 110 * \int_q^1 k^9 - k^{10}\, dk$ *P(H) is at least 0.530184*