Assignment 33

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1 1

$$y = \frac{1}{1+e^{ax+b}}$$
$$\frac{1}{y} = 1 + e^{ax+b}$$
$$ln(\frac{1}{y} - 1) = ax + b$$

The we take the data points and plug them in.

$$[(1, 0.2), (2, 0.25), (3, 0.5)]$$
$$ln(\frac{1}{.2} - 1) = a(1) + b$$
$$ln(4) = a + b$$
$$ln(\frac{1}{.25} - 1) = a(2) + b$$
$$ln(3) = 2a + b$$
$$ln(\frac{1}{.5} - 1) = a(1) + b$$
$$ln(1) = 3a + b$$
$$0 = 3a + b$$

We then turn this system of equations into a matrix equation

$$\begin{bmatrix} ln(4) \\ ln(2) \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

In order to solve for a and b we convert this to the form

$$\frac{1}{\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}} \left[\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} ln(4) \\ ln(2) \\ 0 \end{bmatrix} \right] = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\frac{-ln(4)}{2}}{\frac{3ln(3)}{3} + \frac{4ln(4)}{3}} \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -0.693 \\ 2.215 \end{bmatrix}$$

therefore

$$y = \frac{1}{1 + e^{-0.693x+2.215}}$$

2 2

1.

$$E[aX] = aE[X]$$

$$E[X] = \int_{-\infty}^{\infty} x * p(x)\, dx$$

$$E[aX] = \int_{-\infty}^{\infty} a * x * p(x)\, dx$$

Since a is a constant, we can take pull it out

$$E[aX] = a * \int_{-\infty}^{\infty} x * p(x)\, dx$$

Since

$$E[X] = \int_{-\infty}^{\infty} x * p(x)\, dx$$

$$E[aX] = aE[X]$$

2.

$$\text{Var}[X] = E[X^2] - E[X]^2$$

$$\text{Var}[X] = E[(X - E[X])^2]$$

$$= E[X^2 - 2XE[X] + E[X]^2]$$

$$= E[X^2] - E[2XE[X]] + E[E[X]^2]]$$

Since $E[X]$ is a constant, we can pull it out

$$= E[X^2] - 2E[X]E[X] + E[X]^2$$

$$= E[X^2] - E[X]^2$$