Assignment 29

Nathan Allen

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(a)

$$P(1,17,8,25,3) = \frac{1}{k} * \frac{1}{k} * \frac{1}{k} * \frac{1}{k} * \frac{1}{k} =$$

therefore

$$P(1,17,8,25,3|k) = \begin{cases} \frac{1}{k^5} & k \geq 25 \\ 0 & otherwise \end{cases}$$

(b)

$$\sum_{k=1}^{\infty} c * \frac{1}{k^5} = 1$$

$$c = \frac{1}{\sum_{k=1}^{\infty} c * \frac{1}{k^5}}$$

$$c = 0.964387$$

$$P(k|1,17,8,25,3) = c * P(1,17,8,25,3|k)$$

$$= \frac{0.964387}{k^5}$$

(c) The most likely value of k is 25 since that is the highest number and since $x \in 1, 2, ..., k$ k is the highest number x can be.

(d) The probability that 25 is the upper bound is $\frac{25}{k}$.

(e) The probability that k is less than or equal to 30 is

$$P(25 \leq k \leq 30) = \frac{1}{k} + \frac{1}{k} + \frac{1}{k} + \frac{1}{k} + \frac{1}{k} + \frac{1}{k} =$$

$$\frac{6}{k} =$$

(f)

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(a)

$$\int_c^d \int_a^b k \, dxdy = 1$$

$$\int_c^d bk - ak \, dy = 1$$

$$cbk - dbk - cak + dak = 1$$
$$k * (cb + da - db - ca) = 1$$
$$k = \frac{1}{(b-a)(c-d)}$$

(b)

$$E[X] = \int_c^d \int_a^b x * k \, dxdy =$$
$$\frac{k}{2} * \int_c^d b^2 - a^2 \, dy =$$
$$\frac{k}{2} * (cb^2 - db^2 - ca^2 + da^2) =$$
$$\frac{cb^2 + da^2 - db^2 - ca^2}{2(b-a)(c-d)} =$$
$$\frac{(b+a)(b-a)(c-d)}{2 * (b-a)(c-d)} =$$
$$\frac{b+a}{2}$$

$$E[Y] = \int_c^d \int_a^b y * k \, dxdy =$$
$$\int_c^d by - ay \, dy =$$
$$\frac{k}{2} * (c^2b - d^2b - c^2a + d^2a) =$$

$$\frac{c^2b + d^2a - d^2b - c^2a}{2(b-a)(c-d)} =$$

$$\frac{(c+d)(c-d)(b-a)}{2 * (b-a)(c-d)} =$$

$$\frac{c+d}{2}$$

(c) What the geometric interpretation of the point $(E[X], E[Y])$ in this rectangle would be the center. This is because the estimations are the average, or the midpoint.