

## 24-2

Nathan Reynoso

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### Problem a

Compute the likelihood of the observed outcome if the coin were fair (i.e.  $k=0.5$ ).

$$\begin{aligned}P(\text{HHTTH} | k = 0.5) &= P(\text{H} | k = 0.5) \cdot P(\text{H} | k = 0.5) \cdot P(\text{T} | k = 0.5) \cdot P(\text{T} | k = 0.5) \cdot P(\text{H} | k = 0.5) \\&= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \\&= \frac{1}{32} \\&= 0.03125\end{aligned}$$

### Problem b

Compute the likelihood of the observed outcome if the coin were slightly biased towards heads, say  $k=0.55$ .

$$\begin{aligned}P(\text{HHTTH} | k = 0.55) &= P(\text{H} | k = 0.55) \cdot P(\text{H} | k = 0.55) \cdot P(\text{T} | k = 0.55) \cdot P(\text{T} | k = 0.55) \cdot P(\text{H} | k = 0.55) \\&= 0.55 \cdot 0.55 \cdot 0.45 \cdot 0.45 \cdot 0.55 \\&= 0.55^3 \cdot 0.45^2 \\&= 0.03369\end{aligned}$$

### Problem c

Compute the likelihood of the observed outcome for a general value of  $p$ . Your answer should be a function of  $k$ .

$$\begin{aligned}P(\text{HHTTH} | k) &= P(\text{H} | k) \cdot P(\text{H} | k) \cdot P(\text{T} | 1 - k) \cdot P(\text{T} | 1 - k) \cdot P(\text{H} | k) \\&= k \cdot k \cdot (1 - k) \cdot (1 - k) \cdot k \\&= k^3 \cdot (1 - k)^2\end{aligned}$$

## Problem d

