26-3

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Part 1 Problem a

If you take bus A, what time do you expect to arrive at work? Justify your answer.

The expected arrival time would be 8:30 am. You get this by adding the expected value of x, which is $\frac{0+20}{2}$, in minutes to the base arrival time as well as the time of the ride to work, which is 20 minutes.

Part 1 Problem b

If you take bus B, what time do you expect to arrive at work? Justify your answer.

The expected arrival time is 8:35 am. Once again, you add the expected value of x, which is $\frac{0+10}{2}$, in minutes to the base arrival time as well as the 20 minute ride.

Part 1 Problem c

If you take bus A, what is the probability that you will arrive on time to work? Justify your answer.

The latest the bus can arrive for him to be on time is 8:10, so we need $P(\text{lateness} < 10)$. This is represented as $\int_0^{10} \frac{1}{20} dx$, which comes out to $\frac{1}{2}$.

Part 1 Problem d

If you take bus B, what is the probability that you will arrive on time to work? Justify your answer.

The latest the bus can arrive for him to be on time is 8:10, so we need $P(\text{lateness} = 0)$. This is represented as $\int_0^{0} \frac{1}{10} dx$, which comes out to 0.

Part 2 Problem a

How long do you expect it to take you to come out of the wormhole? Justify your answer.

Since we are given the formula for expected value, being $\frac{1}{\lambda}$, so it would take 0.25 seconds.

Part 2 Problem b

What's the probability of taking longer than a second to come out of the wormhole? Justify your answer.

This is determined by the integral $\int_1^\infty 4e^{-4t}dt$, which comes out to e^{-4}.

Part 2 Problem c

Fill in the blank: the probability of coming out of the wormhole within ___ seconds is 99.999%. Justify your answer.

Within 2.8782 seconds. We get this by setting the percentage to an integral with upper bound as a variable,

$$\begin{aligned} 0.99999 &= \int_0^x 4e^{-4t}dt \\ 0.99999 &= 1 - e^{-4x} \\ x &= 2.8782 \end{aligned}$$

Part 2 Problem d

Your friend says that you shouldn't use the wormhole because there's always a chance that you might get stuck in it for over a day, and if you use the wormhole often, then that'll probably happen sometime within your lifetime. Is this a reasonable fear? Why or why not? Justify your answer by computing the probability that you'll get stuck in the wormhole for over a day if you use the wormhole 10 times each day for 80 years.

The probability that you WON'T get stuck is set up as $\int_0^{86,400} 4e^{-4t}dt$, which is $1 - e^{-345,600}$. Then we raise this to the power of $10 \cdot 365 \cdot 80 = 292,000$, which is $(1 - e^{-345,600})^{292,000}$, which is 1, so the probability that we WILL get stuck is the compliment, which is 0.