

30-1

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Problem a

$$P(T \leq t) = \begin{cases} \frac{1}{16}t^2 & \text{for } 0 \leq t \leq 4 \\ 1 & \text{for } t \geq 4 \end{cases}$$

(a) Find the probability that the job is completed in less than one hour.

$$\begin{aligned} P(T \leq 1) &= \frac{1}{16}(1)^2 \\ &= \frac{1}{16} \end{aligned}$$

(b) Find the probability that the job needs more than 2 hours.

$$\begin{aligned} P(T > 2) &= 1 - P(T \leq 2) \\ &= 1 - \frac{1}{16}(2)^2 \\ &= 1 - \frac{1}{4} \\ &= \frac{3}{4} \end{aligned}$$

(c) Find the probability that $1 \leq T \leq 3$.

$$\begin{aligned} P(1 \leq T \leq 3) &= P(T \leq 3) - P(T \leq 1) \\ &= \frac{9}{16} - \frac{1}{16} \\ &= \frac{1}{2} \end{aligned}$$

Problem b

$$P(T \geq t) = e^{-\frac{t}{5}}, \text{ for all } t \geq 0$$

What is the probability that it breaks down in the third year?

Problem c

$$P(k) = P(\{k\}) = \frac{c}{3^k} \text{ for } k = 1, 2, \dots$$

(a) Find c .

$$1 = P(1) + P(2) + P(3) + \dots$$

$$1 = c\left(\frac{1}{3} + \frac{1}{3^2} + \dots\right)$$

$$1 = c \cdot \frac{1}{2}$$

$$c = 2$$

(b) Find $P(\{2,4,6\})$

$$P(\{2,4,6\}) = P(2) + P(4) + P(6)$$

$$= \frac{2}{9} + \frac{2}{81} + \frac{2}{729}$$

$$= \frac{182}{729}$$

(c) Find $P(\{3,4,5,\dots\})$

$$P(\{3,4,5,\dots\}) = P(3) + P(4) + P(5) + \dots$$

$$= \frac{2}{3^3} \cdot \frac{3}{2}$$

$$= \frac{1}{9}$$