32-2

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November 2020

Problem a

Five cards are dealt from a shuffled deck. What is the probability that the dealt hand contains

(a) *Exactly one ace?*

The probability of the first card pulled being an ace is $\frac{4}{52}$. Then the probabilities that the next 4 cards pulled are all not an ace are $\frac{48}{51}, \frac{47}{50}, \frac{46}{49}, \frac{45}{48}$. When multiplied together we get:

$$\frac{4}{52} \cdot \frac{48}{51} \cdot \frac{47}{50} \cdot \frac{46}{49} \cdot \frac{45}{48} = \frac{32430}{541450} = \frac{3234}{54145}$$

For all other instances (such as the second card drawn being the ace) it is the same calculation, so we end up with:

$$\frac{3234}{54145} \cdot 5 = \frac{3234}{10829} = 0.299$$

(b) *At least one ace?*

To find the probability of getting at least one ace, the easiest way is to get the probability of NOT getting any aces, then subtracting that from 1 to get our probability. The probability of getting NO aces goes as follows:

$$\frac{\binom{48}{5}}{\binom{52}{5}}$$

Since this is the compliment of getting at least 1 ace, the probability of getting at least one ace is:

$$1 - \frac{\binom{48}{5}}{\binom{52}{5}} = 1 - 0.659 = 0.341$$

Problem b

You roll a die 5 times. What is the probability at least one value is observed more than once?

The easiest way to solve this is to find the compliment, which is rolling 5 separate values. The total number of outcomes would be 7776, and desired outcomes is 720. To get total outcomes, each roll has 6 possibilities:

$$6^5 = 7776$$

And the desired outcomes is equal to:

$$6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 = 720$$

Then to get the probability of getting at least one repeat, we subtract the desired outcomes divided by the total outcomes from 1:

$$1 - \frac{720}{7776} = 1 - 0.092 = 0.908$$