36-2

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Problem

Suppose you are a mission control analyst who is looking down at an enemy headquarters through a satellite view, and you want to get an estimate of how many tanks they have. Most of the headquarters is hidden, but you notice that near the entrance, there are four tanks visible, and these tanks are labeled with the numbers 52,30,68,7. So, you assume that they have N tanks that they have labeled with numbers from 1 to N. Your commander asks you for an estimate: with 95% certainty, what's the max number of tanks they have?

First off, we need the likelihood of getting these number from a discrete uniform distribution. This is just

$$\begin{aligned} P(\{52,30,68,7\}|N) &= p_N(52)\cdot p_N(30)\cdot p_N(68)\cdot p_N(7) \\ &= \frac{1}{N}\cdot\frac{1}{N}\cdot\frac{1}{N}\cdot\frac{1}{N} \\ &= \frac{1}{N^4} \end{aligned}$$

These results are taken from the piece wise function

$$p_N(x) = \begin{cases} \frac{1}{N} & x \in \{1,2,...,N\} \\ 0 & x \notin \{1,2,...,N\} \end{cases}$$

This then gives us our likelihood function which is

$$P(\{52,30,68,7\}|N) = \begin{cases} \frac{1}{N^4} & N \geq 68 \\ 0 & \text{otherwise} \end{cases}$$

Then we find our posterior distribution, which gives our constant.

$$\sum_{N=1}^{\infty} c \cdot \begin{cases} \frac{1}{N^4} & N \geq 68 \\ 0 & \text{otherwise} \end{cases} = 1$$

$$\frac{1}{\sum_{N=68}^{\infty} \frac{1}{N^4}} = c$$

$$c = 922741.87$$

Then using our code, we can be 95% sure that N is not 183.