42-2

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Problem a

Let A,B, and C be three events in the sample space S. Suppose we know

$$- A \cup B \cup C = S$$
$$- P(A) = \frac{1}{2}$$
$$- P(B) = \frac{2}{3}$$
$$- P(A \cup B) = \frac{5}{6}$$

(a)Find $P(A \cap B)$.

$$\frac{1}{2} - x + x + \frac{2}{3} - x = \frac{5}{6}$$
$$\frac{3}{6} + \frac{4}{6} - x = \frac{5}{6}$$
$$x = \frac{2}{6}$$

(b)Do A,B, and C form a partition of S?
No, because A and B overlap.
(c)Find $P(C - (A \cup B))$.

$$P(C - (A \cup B)) = P(C - \frac{5}{6})$$
$$= \frac{1}{6}$$

(d)If $P(C \cap (A \cup B)) = \frac{5}{12}$,find $P(C)$.

Problem b

Let X and Y be two independent variables. Suppose that we know Var$(2X - Y) = 6$ and Var$(X + 2Y) = 9$. FInd Var(X) and Var(Y).

$$
\begin{aligned}
Var(X+2Y) &= Var(X)+Var(2Y)+2Cov(X,2Y) \\
9 &= Var(X)+Var(2Y)+2Cov(X,2Y) \\
Var(2X-Y) &= Var(2X)+Var(Y)-2Cov(2X,Y) \\
6 &= Var(2X)+Var(Y)-2Cov(2X,Y)
\end{aligned}
$$

Problem c

(a)Find R_x.

$$
\begin{aligned}
R_x &= x_1, x_2, ... \\
R_x &= 0, 1, 2
\end{aligned}
$$

(b)Find $P(X \geq 1.5)$.

$$
\begin{aligned}
P(X \geq 1.5) &= P(2)+P(3)+... \\
&= \frac{1}{6}+0+... \\
&= \frac{1}{6}
\end{aligned}
$$

(c)Find $P(0 < X < 2)$.

$$
\begin{aligned}
P(0 < X < 2) &= P(1) \\
&= \frac{1}{3}
\end{aligned}
$$

(d)Find $P(X = 0|X < 2)$.

$$
\begin{aligned}
P(X = 0|X < 2) &= \frac{P(X = 0 \cap X < 2)}{P(X < 2)} \\
&= \frac{\frac{1}{2}}{\frac{1}{3}+\frac{1}{2}} \\
&= \frac{6}{10} \\
&= 0.6
\end{aligned}
$$

Problem e

(a)Find $P(A|B)$.

$$\begin{aligned} P(A|B) &= \frac{P(A \cap B)}{P(B)} \\ &= \frac{0.2}{0.35} \\ &= 0.571 \end{aligned}$$

(b)Find $P(C|B)$.

$$\begin{aligned} P(C|B) &= \frac{P(C \cap B)}{P(B)} \\ &= \frac{0.15}{0.35} \\ &= 0.428 \end{aligned}$$

(c)Find $P(B|A \cup C)$.

$$\begin{aligned} P(B|A \cup C) &= \frac{P(B \cap (A \cup C))}{P(A \cup C)} \\ &= \frac{0.25}{0.7} \\ &= 0.357 \end{aligned}$$

(d)Find $P(B|A, C) = P(B|A \cap C)$.

$$\begin{aligned} P(B|A \cap C) &= \frac{P(B \cap (A \cap C))}{P(A \cap C)} \\ &= \frac{0.1}{0.2} \\ &= 0.5 \end{aligned}$$

Problem f

In a factory there are 100 units of a certain product, 5 of which are defective. We pick three units from the 100 units at random. What is the probability that exactly one of the is defective?

$$\begin{aligned} P(2working, 1defective) &= 3 \cdot P(\text{working}) \cdot P(\text{working}) \cdot P(\text{defective}) \\ &= 3 \cdot \frac{97}{100} \cdot \frac{96}{99} \cdot \frac{3}{98} \\ &= \frac{27396}{970200} \\ &= 3 \cdot 0.028 \\ &= 0.084 \end{aligned}$$