Assignment 23

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Part 1

(a)

$$\begin{aligned}\int_{-\infty}^{0} 0\,dx + \int_{0}^{\infty} \lambda e^{-\lambda x}\,dx & \\ &= 0 + -e^{-\lambda x}\big|_{x=0}^{x=\infty} \\ &= 0 - (-1) \\ &= 1\end{aligned}$$

(b)

$$\begin{aligned}\int_{0}^{1} \lambda e^{-\lambda x}\,dx & \\ &= -e^{-\lambda x}\big|_{x=0}^{x=1} \\ &= -e^{-\lambda} - (-1) \\ &= 1 - e^{-\lambda}\end{aligned}$$

(c)

$$\begin{aligned}\int_{-\infty}^{0} 0\,dx + \int_{0}^{\infty} x\lambda e^{-\lambda x}\,dx & \\ &= 0 - e^{-\lambda x}x - \frac{1}{\lambda}e^{-\lambda x}\big|_{x=0}^{x=\infty} \\ &= 0 - \left(-\frac{1}{\lambda}\right) \\ &= \frac{1}{\lambda}\end{aligned}$$

(d)

$$\int_{-\infty}^{\infty} \left(x - \frac{1}{\lambda}\right)^2 * p(x)\, dx$$

$$\begin{aligned}
&= \int_{-\infty}^{0} \left(x - \frac{1}{\lambda}\right)^2 * 0\, dx + \int_{0}^{\infty} \left(x - \frac{1}{\lambda}\right)^2 * \lambda e^{-\lambda x}\, dx \\
&= -e^{-\lambda x}x^2 - \frac{1}{\lambda^2}e^{-}\Big|_{x=0}^{x=\infty} \\
&= 0 - \left(-\frac{1}{\lambda^2}\right) \\
&= \frac{1}{\lambda^2}
\end{aligned}$$

Part 2

(a)

$$\begin{aligned}
\int_a^b k\, dx &= 1 \\
kx\Big|_{x=a}^{x=b} &= 1 \\
bk - ak &= 1 \\
(b-a)k &= 1 \\
k &= \frac{1}{(b-a)}
\end{aligned}$$

(b)

$$\int_{-\infty}^{a} p_\lambda x\, dx = 0, x = 0 \text{when} x \notin [a, b]$$

$$\int_{b}^{\infty} p_\lambda x\, dx = 0, x = 0 \text{when} x \notin [a, b]$$

$$\begin{aligned}
\int_a^b p_\lambda x\, dx &= \int_a^b k\, dx \\
&= \int_a^b \frac{1}{(b-a)}\, dx \\
&= \frac{x}{(b-a)}\Big|_{x=a}^{x=b} \\
&= \frac{b}{(b-a)} - \frac{a}{(b-a)} \\
&= \frac{b-a}{b-a} \\
&= 1
\end{aligned}$$

$$P(X \leq x) = \begin{cases} 0 & x < a \\ 1 & a \leq x \leq b \\ 0 & b < x \end{cases}$$

(c)

$$\int_a^b \frac{x}{(b-a)}\,dx$$

$$\begin{aligned}
&= \frac{x^2}{2(b-a)}\Big|_{x=a}^{x=b} \\
&= \frac{b^2}{2(b-a)} - \frac{a^2}{2(b-a)} \\
&= \frac{b^2-a^2}{2(b-a)} \\
&= \frac{b+a}{2}
\end{aligned}$$

(d)

$$\int_a^b (x-\frac{b+a}{2})^2 k\,dx$$

$$\begin{aligned}
&= k\int_a^b (x-\frac{b+a}{2})^2\,dx \\
&= \frac{1}{(b-a)}(\frac{x^3}{3} - \frac{bx^2}{2} - \frac{ax^2}{2} + \frac{(b+a)^2}{4})\Big|_{x=a}^{x=b} \\
&= (\frac{1}{(b-a)}(\frac{b^3}{3} - \frac{b^3}{2} + \frac{ab^2}{2} * \frac{(b+a)^2}{4})) - (\frac{1}{(b-a)}(\frac{a^3}{3} - \frac{ba^2}{2} + \frac{a^3}{2} * \frac{(b+a)^2}{4})) \\
&= \frac{-3a^5 - 6ba^4 - 8a^3 + 6b^3a^2 + 12ba^2 + 3b^4a - 4b^3}{24\,(b-a)} \\
&= \frac{(b-a)^2}{12}
\end{aligned}$$