Assignment 27

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27-3

(a)

$$k^9 \cdot (1 - k^1) = k^9 - k^{10}$$

(b)

$$\begin{aligned}
P(k|\text{HHHHT HHHHH}) &= c \cdot P(\text{HHHHT HHHHH}|k) \\
P(k|\text{HHHHT HHHHH}) &= c(k^9 - k^{10})
\end{aligned}$$

$$\begin{aligned}
\int_0^1 c(k^9 - k^{10})\, dk &= 1 \\
c \cdot \int_0^1 k^9 - k^{10}\, dk &= 1 \\
c\left(\frac{k^{10}}{10} - \frac{k^{11}}{11}\Big|_{x=0}^{x=1}\right) &= 1 \\
c \cdot \frac{1}{110} &= 1 \\
c &= 110
\end{aligned}$$

(c)

$$\begin{aligned}
\int_{0.5}^1 p(k)\, dk \\
&= \int_{0.5}^1 k\, dk \\
&= \int_{0.5}^1 \frac{1}{(1-0)}\, dk \\
&= \int_{0.5}^1 1\, dk \\
&= k\Big|_{x=0.5}^{x=1} \\
&= 1 - 0.5 \\
&= 0.5
\end{aligned}$$

(d)

$$\int_{0.5}^{1} 110(k^9 - k^{10})\,dk$$

$$= 110\left(\frac{k^{10}}{10} - \frac{k^{11}}{11}\Big|_{x=0}^{x=1}\right)$$
$$= 0.994$$

(e)

The typical probability of the coin is 50%, or 0.5. However, with the one sample given that is heavily heads biased, it makes sense that the probability shoots up to 99%.

(f)

If $P(k|\text{HHHHT HHHHHH}) = 110k^9 - 110k^{10}$, then $P' = 990k^8 - 1100k^9$ When $P' = 0$, we get that $k = 0.9$, which is its maximum.

(g)

This makes sense because out of the 10 flips, 9 landed heads, which is equal to a 0.9 probability.

(h)

$$\int_{0.85}^{0.95} 110(k^9 - k^{10})\,dk$$

$$= 110\left(\frac{k^{10}}{10} - \frac{k^{11}}{11}\Big|_{x=0.85}^{x=0.95}\right)$$
$$= 0.40591$$

(i)

$$\int_{a}^{1} 110(k^9 - k^{10})\,dk = 0.99$$
$$110\left(\frac{k^{10}}{10} - \frac{k^{11}}{11}\Big|_{x=a}^{x=1}\right) = 0.99$$
$$110\left(\frac{1}{10} - \frac{1}{11} - \frac{a^{10}}{10} + \frac{a^{11}}{11}\right) = 0.99$$
$$a = 0.5302$$