Assignment 35

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35-1

(a)

$$P(1, 17, 8, 25, 3|k) = \begin{cases} \frac{1}{k^5} & k \geq 25 \\ 0 & otherwise \end{cases}$$

(b)

$$\sum_{k=1}^{\infty} c \cdot P(1, 17, 8, 25, 3|k) = 1$$
$$c \cdot \sum_{k=1}^{\infty} P(1, 17, 8, 25, 3|k) = 1$$
$$c = 1443199.78322$$

$$\sum_{k=1}^{\infty} c \cdot P(k|1, 17, 8, 25, 3) = \begin{cases} \frac{1443199.78322}{k^5} & k \geq 25 \\ 0 & otherwise \end{cases}$$

(c)

The most probable value is k = 25. This is due to the fact that 25 is the lowest possible value that would not result in a probability of 0. This means that when k = 25, there are the least amount of possible values other than 25 that can be picked.

(d)

$$\frac{1443199.78322}{k^5} \rightarrow \frac{1443199.78322}{25^5} = 0.147783658 = 14.78\%$$

(e)

$$P(25 \leq k \leq 30) = \sum_{k=25}^{30} \frac{1443199.78322}{k^5} = 0.58343 = 58.3\%$$

(f)

$$P(25 \leq k \leq n) = \sum_{k=25}^{n} \frac{1443199.78322}{k^5} = 0.95$$

$$n = 53$$

35-2

(a)

$$\begin{aligned}
\int_c^d \int_a^b p(x,y)\,dx\,dy &= 1 \\
\int_c^d \int_a^b k\,dx\,dy &= 1 \\
\int_c^d \left(kx\Big|_{x=a}^{x=b} \right) dy &= 1 \\
\int_c^d (bk - ak)\,dy &= 1 \\
k \cdot \int_c^d (b-a)\,dy &= 1 \\
k \cdot \left(y(b-a)\Big|_{x=c}^{x=d} \right) &= 1 \\
k \cdot (d(b-a) - c(b-a)) &= 1 \\
k &= \frac{1}{(d-c)(b-a)}
\end{aligned}$$

(b)

$$\begin{aligned}
E[X] &= \int_c^d \int_a^b x \cdot p(x,y)\,dx\,dy \\
&= \int_c^d \int_a^b x \cdot \frac{1}{(d-c)(b-a)}\,dx\,dy \\
&= \int_c^d \left(\frac{x^2}{2(d-c)(b-a)}\Big|_{x=a}^{x=b} \right) dy \\
&= \int_c^d \left(\frac{b^2 - a^2}{2(d-c)(b-a)} \right) dy \\
&= \int_c^d \left(\frac{(b-a)(b+a)}{2(d-c)(b-a)} \right) dy \\
&= \int_c^d \left(\frac{b+a}{2(d-c)} \right) dy \\
&= y \cdot \left(\frac{b+a}{2(d-c)} \right) \Big|_{y=c}^{y=d} \\
&= \frac{d(b+a) - c(b+a)}{2(d-c)} \\
&= \frac{(d-c)(b+a)}{2(d-c)} \\
&= \frac{b+a}{2}
\end{aligned}$$

$$
\begin{aligned}
E[Y] &= \int_c^d \int_a^b y \cdot p(x,y)\,dx\,dy \\
&= \int_c^d \int_a^b \frac{y}{(b-a)(d-c)}\,dx\,dy \\
&= \int_c^d \left(\frac{y}{(b-a)(d-c)} x\Big|_{y=c}^{y=d} \right) dy \\
&= \int_c^d \frac{y(b-a)}{(b-a)(d-c)}\,dy \\
&= \int_c^d \frac{y}{d-c}\,dy \\
&= \frac{y^2}{2(d-c)}\Big|_{y=c}^{y=d} \\
&= \frac{d^2}{2(d-c)} - \frac{c^2}{2(d-c)} \\
&= \frac{d^2-c^2}{2(d-c)} \\
&= \frac{(d-c)(d+c)}{2(d-c)} \\
&= \frac{c+d}{2}
\end{aligned}
$$

(c)

The geometric interpretation is the midpoint bounded by $[a,b] \times [c,d]$ of the rectangle.