Assignment 42

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a)

(a)

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) = \frac{1}{2} + \frac{2}{3} - \frac{5}{6} = \frac{1}{3}$$

(b)

They do not form a partition, though the union between all of them make up S, there is overlap and A and B are not disjoint, so it fails to make a partition.

(c)

Since $A \cup B \cup C = S$, there is no data outside of A, B and C, and therefore the 3 events combined equal to 1. This means that $P(C - (A \cup B))$ is the area exclusive to C, which is $A \cup B$ subtracted from the entire circle:

$$1 - \frac{5}{6} = \frac{1}{6}$$

(d)

Since $P(C \cap (A \cup B))$ is the area that C shares with A and B and $P(C - (A \cup B))$ is the area exclusive to C, adding these two values together will give us the entirety of $P(C)$:

$$\frac{5}{12} + \frac{1}{6} = \frac{7}{12}$$

b)

$$\begin{aligned}
Var[X+2Y] &\rightarrow \\
Var[X] + Var[2Y] + 2Cov[X, 2Y] &= 9 \\
Var[X] + 4Var[Y] &= 9 \\
Var[X] &= 9 - 4Var[Y] \\
\\
Var[2X - Y] \rightarrow Var[2X + (-Y)] &\rightarrow \\
Var[2X] + Var[-Y] + 2Cov[2X, -Y] &= 6 \\
4Var[X] + Var[Y] &= 6 \\
Var[Y] &= 6 - 4Var[X] \\
\\
Var[X] &= 9 - 4Var[Y] \\
Var[X] &= 9 - 4(6 - 4Var[X]) \\
Var[X] &= -15 + 16Var[X] \\
-15Var[X] &= -15 \\
Var[X] &= 1 \\
\\
Var[Y] &= 6 - 4Var[X] \\
&= 6 - 4(1) \\
Var[Y] &= 2
\end{aligned}$$

c)

(a)

$$R_X = \{0, 1, 2\}$$

(b)

$$P(X \geq 1.5) = \frac{1}{3}$$

(c)

$$P(0 < X < 2) = \frac{1}{6}$$

(d)

$$P(X = 0 | x < 2) = \frac{\frac{1}{2}}{\frac{1}{3} + \frac{1}{2}} = \frac{3}{5}$$

d)

$$P(z) = \begin{cases} \frac{1}{36} & z = 5 \\ \frac{1}{18} & z = 4 \\ \frac{1}{12} & z = 3 \\ \frac{1}{9} & z = 2 \\ \frac{5}{36} & z = 1 \\ \frac{1}{6} & z = 0 \\ \frac{5}{36} & z = -1 \\ \frac{1}{9} & z = -2 \\ \frac{1}{12} & z = -3 \\ \frac{1}{18} & z = -4 \\ \frac{1}{36} & z = -5 \\ 0 & otherwise \end{cases}$$

$$R_X = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$$

e)

(a)

$$P(A|B) = \frac{0.2}{0.1 + 0.05 + 0.1 + 0.1} = \frac{0.2}{0.35} = 0.5714$$

(b)

$$P(C|B) = \frac{0.15}{0.35} = 0.4285$$

(c)

$$A \cup C = (0.2 + 0.1 + 0.1 + 0.1) + (0.15 + 0.05 + 0.1 + 0.1) - (0.1 + 0.1) = 0.7$$

$$P(B|A \cup C) = \frac{0.25}{0.7} = 0.3571$$

(d)

$$P(B|A \cap C) = \frac{0.1}{0.2} = 0.5$$

f)

$$P = \frac{\frac{5!}{4!\cdot 1!} \cdot \frac{95!}{93!\cdot 2!}}{\frac{100!}{97!\cdot 3!}} = \frac{5 \cdot 4465}{161700} = 0.1380 = 13.8\%$$